Towards Half-Precision Computation for Complex Matrices: A Case Study for Mixed-Precision Solvers on GPUs

Ahmad Abdelfattah, Stan Tomov, and Jack Dongarra

Innovative Computing Laboratory
University of Tennessee, Knoxville

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INNOVATIVE COMPUTING LABORATORY

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Outline

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1. Introduction
2. Matrix Multiplication with Half-complex Precision
3. Mixed-precision Factorization and Solve
4. Final Performance
5. Conclusion

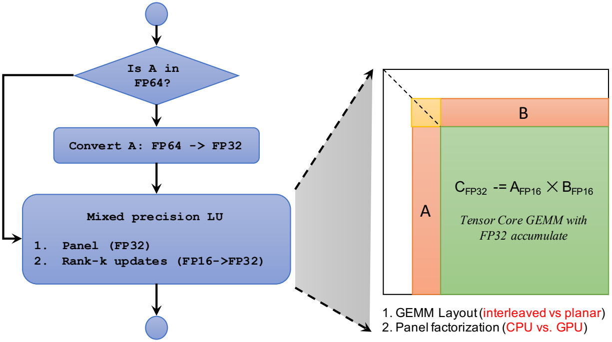
What are trying to solve?

- Solve a linear system of equation ($Ax = b$)
- A is a general square matrix in single-complex precision
- Use half-precision to accelerate the solution (mixed-precision solver)
  - Accuracy is recovered using Iterative Refinement (IR)
- Similar algorithm to
  - *Carson and Higam: Accelerating the Solution of Linear Systems by Iterative Refinement in Three Precisions, SIAM SISC, 2018*
  - *Haidar et al.: Harnessing GPU Tensor Cores for Fast FP16 Arithmetic to Speed Up Mixed-precision Iterative Refinement Solvers, SC'18*
- No native support for half-complex computation

This work is part of the MAGMA library

What are the steps?

1. **Mixed-precision LU factorization with partial pivoting**
2. Iterative refinement using preconditioned GMRES

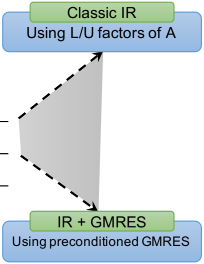


1. GEMM Layout (interleaved vs planar)
2. Panel factorization (CPU vs. GPU)

What are the steps?

1. Mixed-precision LU factorization with partial pivoting
2. **Iterative refinement using preconditioned GMRES**

(1) Compute residual	$r = b - Ax$
(2) Solve for c	$Ac = r$
(3) Update solution	$x_{i+1} = x_i + c$
(4) Repeat 1-3 until converged	



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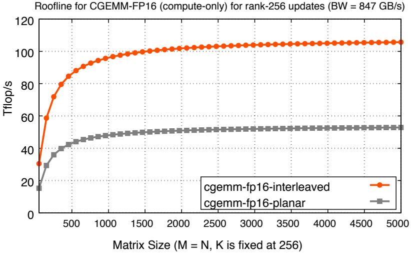
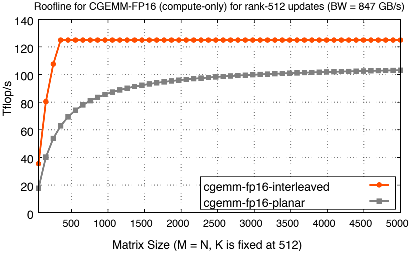
Mixed-precision Matrix Multiplication

- $C_{FP32} = C_{FP32} - A_{FP16} \times B_{FP16}$
- A and B are originally in FP32, but are converted to half-complex
- No native support for half-complex kernels
  - cuBLAS provides only real arithmetic FP16 kernels
  - cuBLASLt and CUTLASS suggest using split-complex kernels using planar layout
- But interleaved layout is a better alternative
  - Used by almost all linear algebra algorithms
  - Better operational intensity (i.e. flops/bytes ratio)

Mixed-precision Matrix Multiplication

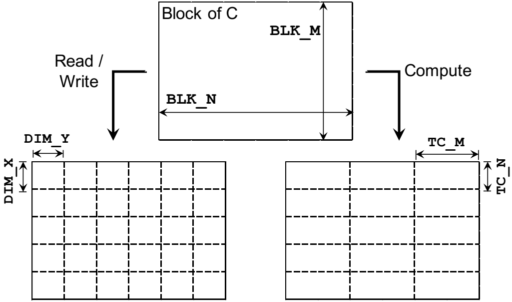
- Goal: `CGEMM` kernel with FP16 acceleration
- Using cuBLAS (planar-layout)
  - Four calls to `cublasGemmEx`: arith. intensity = $\frac{mnk}{4mn+k(m+n)}$
  - Overhead of splitting and merging real/imaginary components
- Using a new MAGMA kernel (interleaved-layout)
  - One kernel call: arith. intensity = $\frac{2mnk}{4mn+k(m+n)}$
  - No overheads





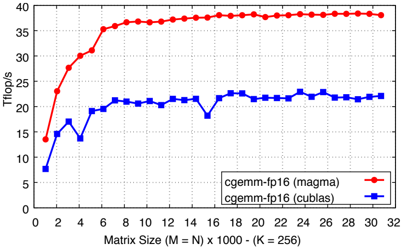
MAGMA's New Kernel (CGEMM-FP16-Interleaved)

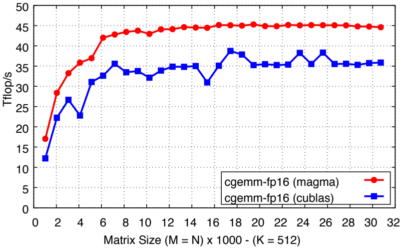
- Input: *A* and *B* in half-complex precision (`half2`)
- Input/Output: *C* in single-complex precision
- An abstraction layer over the Tensor Cores (TCs)
  - Using `WMMA` device routines to manage TCs
  - Split and merge in shared memory
- Double-sided recursive blocking + auto-tuning



Performance of CGEMM-FP16

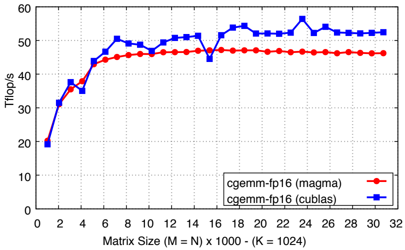
- Tested on Tesla V100-PCIe GPU, CUDA 10.1
- 2 advantages for MAGMA: arithmetic intensity and splitting/merging overhead
- 70% better than cuBLAS ($k = 256$), 24% if $k = 512$





Performance of CGEMM-FP16 “cont.”

- Tested on Tesla V100-PCIe GPU, CUDA 10.1
- MAGMA loses the advantage for $k \geq 800$
- Summary of results
  - Blocking size ≤ 800, use MAGMA
  - Otherwise, use cuBLAS

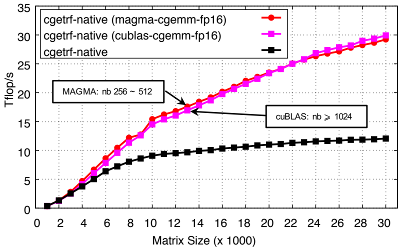


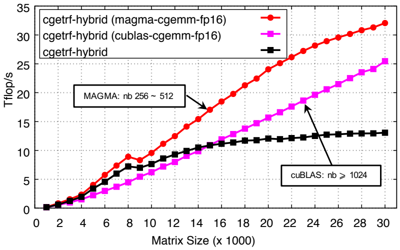
Let’s test both!

Outline

Performance of the mixed-precision LU factorization

- Panel factorization: CPU (hybrid) vs. GPU (native)?
- Larger *nb* means more time spent in single-complex precision (i.e. the panel)
- Use hybrid-LU only if the matrix is larger than 22*k* (with MAGMA CGEMM-GP16)

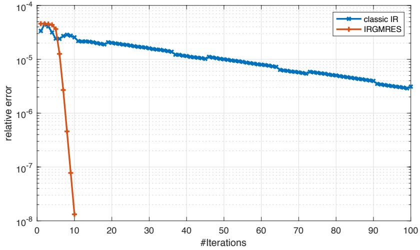




Performance of the mixed-precision LU factorization using native (left) and hybrid (right) executions. Results are shown on a 20-core Haswell CPU and a Tesla V100 GPU. MAGMA is built using CUDA 10.1 and MKL 2018.0.1

Classic IR vs. IR + GMRES

- GMRES is more stable for solving the correction equation ($Ac = r$)
- GMRES is preconditioned by the low precision factors of A
- Much faster conversion than classic IR
- Based on FGMRES implementation by Yousef Saad (ZITSOL)



Convergence history of both `IR` *and* `IRGMRES` *on a matrix of size 20k.* $k_\infty(A) = 10^5$. *Clustered distribution of singular values* ($\sigma_i = 1, 1, \cdots, \frac{1}{k_\infty(A)}$).

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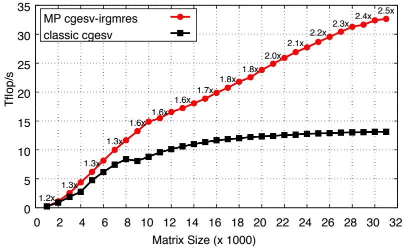
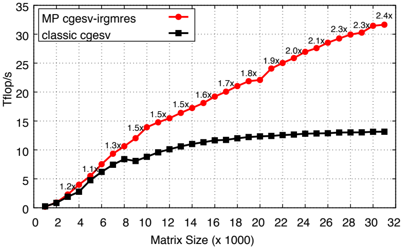
Final Performance: Mixed-precision CGESV-IRGMRES

- Performance is up to 2.5×, but depends on the matrix properties





System: 20-core Intel Haswell CPU, Tesla V100 GPU - using MKL 2018.0.1 and CUDA 10.1

Left: Diagonally dominant matrices. $k_\infty(A) \leq 10^2$.

Right: Matrices with positive eigenvalues and arithmetic distribution of singular values $(\sigma_i = 1 - (\frac{i-1}{n-1})(1 - \frac{1}{k_\infty(A)}))$, $k_\infty(A) \approx 4.3e{+}5$

Final Performance: Mixed-precision CGESV-IRGMRES

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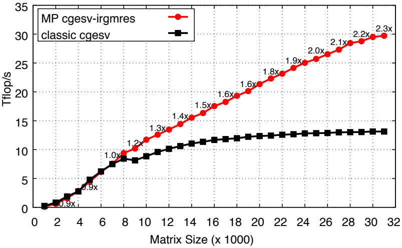

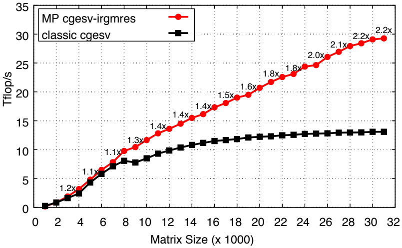

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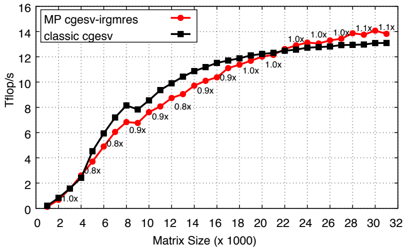
Left: Positive eigenvalues and logarithmic uniform distribution of singular values ($\log(\sigma_i)$ uniform between $\log(\frac{1}{k_\infty(A)})$ and $\log(1)$), $k_\infty(A) \approx 1e{+}5$

Right: Clustered singular values ($\sigma_i = 1, 1, \cdots, \frac{1}{k_\infty(A)}$), $k_\infty(A) \approx 4.3e{+}4$

Sometimes it does not pay off

- The refinement steps can consume all the performance advantage of the factorization



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Arithmetic distribution of singular values $(\sigma_i = 1 - (\frac{i-1}{n-1})(1 - \frac{1}{k_\infty(A)}))$, $k_\infty(A) \approx 4.3e{+}4$.

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Summary

1. FP16 arithmetic is not only for machine learning
2. New family of mixed-precision linear solvers
3. Half-complex precision accelerates single-complex systems by factors up to 2.5$\times$
4. Next: Double-complex systems, matrix scaling, other half-complex kernels

- https://icl.utk.edu/magma/
- Release expected by Spring 2020

Thank You!