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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Two Perspectives on Recent Books on Ordinary Differential Equations

Formal Power Series and Linear Systems of Meromorphic Ordinary Differential Equations. *By Werner Balser.* Springer-Verlag, New York, 2000. $49.95. xviii+299 pp., hardcover. ISBN 0-387-98690-1.

Ordinary Differential Equations with Applications. *By Carmen Chicone.* Texts in Applied Mathematics. Volume 34. Springer-Verlag, New York, 1999. $59.95. xvi+561 pp., hardcover. ISBN 0-387-98535-2.

Basic Theory of Ordinary Differential Equations. *By Po-Fang Hsieh and Yasutaka Sibuya.* Springer-Verlag, New York, 1999. $59.95. xii+468 pp., hardcover. ISBN 0-387-98699-5.

First Reviewer. First, I would like to give some information about each book, then I will compare and discuss them and give my personal appreciation of them.

Balser (B) is intended as a reference book in the special area of the analytic theory of linear meromorphic ordinary differential equations (ODEs) as well as an introduction to this topic for students who want to work in this or adjacent fields. After an introduction containing several examples motivating the study of divergent formal solutions of ODEs, the basic properties of solutions and singularities of the first kind are briefly studied. Then the following topics are discussed in depth: formal solutions, asymptotic expansions, Borel- and Laplace-transforms, Gevrey asymptotics, summability, the Stokes' phenomenon, and multisummability as well as tools required such as the Cauchy–Heine transform and Ecalle's acceleration operators. Finally, related topics as the Riemann–Hilbert and reduction problems are briefly mentioned, as are applications of the methods to adjacent areas (in particular nonlinear ODEs, difference equations, and singular perturbations).

Chicone (C) is intended as a text for the graduate level. In the first chapter, the basic theory of ODEs is covered—in contrast to the "logical" order of the theory, some results are introduced that are treated in depth in later chapters. The following chapters treat linear systems and stability theory, applications (real-life applications from physics!), invariant manifolds, persistence of periodic solutions under perturbations, Melnikov's method and homoclinic orbits, and averaging and bifurcation theory. Applications are given throughout the book, often as motivation for the theory.

Hsieh and Sibuya (HS) is also intended as a textbook for the graduate level. The first chapters cover the basic theory of ODEs—here an in-depth study of nonuniqueness is exceptional. Then, singularities of the first kind are treated (using S–N decompositions), as are boundary value problems, Lyapunov-type numbers, stability

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theory including perturbation theory of spiral points, etc., autonomous systems, second order linear equations, asymptotic expansions (including Gevrey theory, also in a parameter), and singularities of the second kind.

Clearly, (B) cannot be compared to the other two books; its purpose as a reference and introduction to some specialized research is too different from the purpose of the others. (B) is very welcome, as much of the material cannot be easily found in books published in English (except maybe in Balser's earlier lecture notes, which are no longer available). The extensive bibliography containing many recent articles in this field is very valuable. (B) could also be used in advanced special topics courses for doctoral students, which are common in Germany and France.

The books (C) and (HS) are both intended as textbooks. Their first chapters could be studied in an introductory course, and some of the following chapters could be studied in a subsequent course. It is impossible to cover the content of either of the two books in a one-year course. The title of (HS) is therefore somewhat misleading—the book contains much more than the basic theory of ODEs. Both books are rigorous in their proofs and contain many details and supplementary material such as analysis in Banach spaces (C), or complete proofs of the existence of a Jordan canonical form or Gevrey asymptotics (HS). In using them as textbooks, I would add some more details here and there, depending on the level of the students. Both books fail to indicate topics they do not cover—it would be nice if (C) would mention ODEs in the complex domain (not even Bessel's equation is present) or boundary value problems, and it would be nice if (HS) mentioned applications of ODEs and averaging and bifurcation theory.

While some (nonbasic) subjects are covered in both books (such as stability theory, Poincaré–Bendixson, and the existence of periodic solutions), some topics are treated only in (C) (in particular, homoclinic orbits/Melnikov theory, averaging, bifurcation, and applications to physics), while others are found only in (HS) (boundary value problems, asymptotic expansions, and singularities of ODEs in the complex domain). Some topics, such as singular boundary value problems, are absent from both books. It is no surprise then that Chicone's research interests also contain dynamical systems, differential geometry, and applied mathematics, whereas Hsieh and Sibuya are mostly interested in asymptotic expansions and singularities of ODEs.

This is, however, only one aspect that shows the differences between the books. (HS) reminds me of the classical ODE textbook by Coddington and Levinson in its content (far from identical, however, and many topics are treated in a new way) as well as in its style. This was, as they state, intended by the authors. (C), in much of its mathematical contents, reminds me of Hale's book on ODEs; I do not know, however, any ODE book that covers both theory and applications in the way (C) does; furthermore, its style is more conversational than that of (HS).

I very much appreciate that (C) treats both applications of ODEs to not-simplified problems and—rigorously and in detail—the theory required to do this. I very much like the first chapter, which often introduces concepts and theorems without proving them immediately—the "logical" arrangement of a theory delays the appearance of powerful theorems and thus their possible application; also, I think, it is very motivating for students to know that useful, deep results will be proved later on.

I admire the powerful, often optimal theorems and the elegant, concise proofs of (HS) that make it both a reference book for all who teach ODEs and an initiation into the theory.

Which of the textbooks I would use in my course on ODEs would depend mostly on the students. If sufficiently many students with interests in applications or in geometry were present, I would use (C) or, more precisely, a presentation in the style of (C); for a more theoretical course, I would use (HS) and a "logical approach." My choice of topics for a second semester (with a "general" audience) would include boundary value problems as well as averaging and some applications, i.e., topics from both books. In any case, I would recommend (C) and (HS) and other books on ODEs as sources of additional interesting material. In order to prepare future doctoral students, I would use the later chapters of (HS) and (B). Yes, I also work in asymptotic expansions and study singularities of ODEs.

To sum up, all three books are welcome additions to the literature on ODEs: (B) as an advanced research work on a special topic, (C) as a bridge between rigorous mathematical theory and real-world applications, and (HS) as a reference for the theory. I will use all three of them.

Reinhard Schäfke
University of Strasbourg

Second Reviewer. It seems like a paradox, but the fact that we are drowning in data caused by ever intenser multimedia communication makes *personal contacts* and *books* more and more important. Just because of this flood tide there is a growing need of good road signs, advice, and guide books. Digesting the literature has become impossible, except in a global way, and more than ever we should have excellent books.

The subject of the book by Balser is linear equations with complex coefficients. As we know, series expansions present difficulties in the case of singularities, in particular near irregular singularities. In this book one systematically looks at formal series expansions, generally divergent near irregular singularities, to apply the concepts of summability and multisummability. These concepts were developed by Jean Ecalle, and the last twenty years have seen the addition of many technical aspects and generalizations. In fact this was the main reason to present here a rather complete theory of power series expansions for linear ODEs.

There are a few pages on applications to other areas such as nonlinear systems, singular perturbations, and PDEs. In fact singular perturbations of ODEs and expansions for PDEs have in this context a lot in common as the small parameter plays the part of an additional variable. These notes on new applications are too short to be convincing. On the other hand, in these fields so many fundamental problems remain with respect to asymptotics that new theory should certainly be considered.

The classic on ODEs is the book by Coddington and Levinson [1], and to this day it plays a part in research and as a standard reference. In the preface to their book, Hsieh and Sibuya state that the classic by Coddington and Levinson "is used as a role model." So, this raises high expectations.

Their text can be divided into four parts: fundamental existence and uniqueness results; the basics of linear equations; three chapters on nonlinear equations; and material on power series solutions.

On nonlinear systems there are basic results on stability by linearization (Poincaré–Lyapunov-type theorems) and invariant manifolds. Other topics like Lyapunov's direct method are given too little space. The chapter on quasi-linear second-order equations was an important research topic fifty years ago—it still figures prominently in teaching—but I can think of many other more important subjects to replace it in a basic text such as, for instance, normalization, bifurcation theory and the fundamentals of dynamical systems in particular, periodic solutions, and invariant manifolds.

The central part of the book is really the exhaustive treatment of linear systems. It reflects the developments of the last decades with, for instance, discussions of S-N decomposition of a linear operator instead of using the Jordan normal form, its use for singularities as proposed by Gérard and Levelt, and asymptotic expansions both in the sense of Poincaré and by Gevrey asymptotics. There is some overlap here with Balser's book. Very useful is the treatment of linear systems with periodic coefficients, both in the general Floquet case and in the Hamiltonian context.

This is a valuable text. However, do not throw away your Coddington and Levinson, which contains gems such as the generalized Gronwall lemma and interesting additional chapters such as the one on torus equations.

Carmen Chicone's book is on ODEs, dynamical systems, and applications. It should be compared with a classic such as Guckenheimer and Holmes [3] or with Glendinning [2] and Verhulst [4].

The author prepares the ground with topics such as manifolds, Poincaré–Bendixson theory for periodic solutions, implicit functions, and existence and uniqueness—all this at a fast pace and without giving all the details. This could be heavy going for beginners.

Around 50 pages are spent on linear systems with attention to stability, Lyapunov exponents, and Floquet theory, and contain useful material and interesting examples. The advanced chapters are concerned with hyperbolicity, including an extensive account of the Hartman–Grobman theory, continuation of periodic solutions (interesting but again not easy for beginners), homoclinic orbits-Melnikov-chaos, averaging methods (brief but nice), and bifurcation theory.

The didactical style of the book is somewhat uneven. Some topics—for instance, bifurcation theory—are very well introduced; other ones—such as the Lyapunov–Schmidt method—are presented in a rather concise way. The applications are a separate aspect. The author has put together an unusually nice collection of interesting problems: perturbed Keplermotion, coupled pendula, the Fermi–Pasta–Ulam chain, and traveling waves. A lot of attention is also paid to "where do the problems come from," which is usually classical mechanics and PDEs.

The three books on ODEs considered here are not elementary but are certainly within reach of a student with an understanding of basic analysis and linear algebra. The Hogwarts-level is intermediate.

The book by Balser fills a niche by describing a special research topic—expansions of near singularities of linear ODEs. The text by Hsieh and Sibuya, on the other hand, comprises an important and fairly complete account of the basics of ODEs and linear systems. This is very useful.

As an applied mathematics text on linear and nonlinear equations, the book by Chicone is written with stimulating enthusiasm. It will certainly appeal to many students and researchers.

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Ferdinand Verhulst
University of Utrecht

Evolutionary Algorithms: The Role of Mutation and Recombination. *William M. Spears*, Springer-Verlag, New York, 2000. $39.95. xiv+222 pp., hardcover. ISBN 3-540-66950-7.

This medium-size, very readable book is a welcome addition to the exploding literature on genetic algorithms (GAs). It is self-contained and can serve as a reference for discrete probabilistic static and dynamic analysis of recombination and mutation aspects of GAs, as well as an introduction to the active research in the field, to which the author is a prominent contributor. Thus, there are many well-documented references and connections to other work, critiques, and clearly defined personal positions. Among other clearly stated explanations on different terminology, the reader will learn that GAs are only a subspecies (albeit the main one) of the evolutionary algorithms (EAs) (the others are evolutionary programming and evolution strategies; see Chap. 1). The text repeats arguments when necessary, making many of the chapters suitable for independent reading. It is, in general, a very well structured text.

The book assumes little background in the subject. The mathematical prerequisites are elementary discrete probability (including the definition of a Markov chain), combinatorics, and linear algebra (only for Chap. 13). The computer science prerequisites assume only an understanding of the idea of an algorithm and simple data structures as a basis for a program, and complexity analysis, in order to justify designing different algorithms for the same problem (as in the novel aggregation algorithm for Markov chains in Chap. 13, whose applications could reach into other areas of computer-based process simulation). It does assume the level of maturity of a senior math or computer science undergraduate or graduate student. But, most importantly, it assumes the curiosity and imagination on the part of the reader to answer the question, "Why are all these analyses useful?", because the book contains no examples of applications outside the field.

I first learned about GAs in the early 1990s. It is reassuring to find out that models and techniques developed in the field of GAs are successfully employed elsewhere (genetics, from where many of the terms are borrowed, machine learning, neural networks, symbolic computation, approximate solutions to computer intractable problems). The text analyzes, from a discrete probabilistic viewpoint, the influence mutation and recombination have on the convergence of GAs. Computing the expected number of individuals in a common structure does this. Essentially, one can construct or destruct higher order structures (represented as binary strings) from lower order ones using recombination (exchanging parts of the strings representing the two parents) and mutation (modifying parts of the strings according to a given probability). The analyses are repeatedly (Chaps. 3–9 for the static case and Chaps. 9–12 for the dynamic case) well explained, rigorously performed, and accompanied by suggestive graphs of empirical simulations validating the conclusions of the formulas. They begin by analyzing the population probability distributions for the recombination and mutation operators separately, then combined, and then for some particular cases: uniform recombination, independence, and identicalness. Finally, characteristics such as exploratory power and positional and distributional bias (which essentially measure different degrees of merit for recombination and mutation) are discussed. In this respect the book is an example of an honest effort to build an objective evaluation testbed for EAs. It contains very valuable information for practitioners in the field. The results on comparison rates for reaching Robbins (uniform) equilibrium for different recombination (mutation) operators offer good hints on choosing appropriate parameter values for the data structures in the simulation programs; see Chap. 10.

The book is a bit dry because there are very few examples or applications from outside the field (with the exception of the selection function from biology to drive the empirical analyses in Chap. 11 and the Boolean satisfiability problem from complexity theory as a testbed problem generator in Chap. 2). The graphs could be made more intuitive but they are well accompanied by explanations. It would also be interesting to see extensions of these results to the continuum case. In particular, it is not clear how the frameworks will translate. Admittedly, the author leaves this as a future work direction.

I learned from this text. It was refreshing to see rigor and attention to detail brought into a field where theoretical investigations have too often been plagued by handcrafted scenario assumptions. It may well mark a long-sought turning point.

Eugen Ardeleanu
Microsoft Corporation

The Mathematics of Measurement: A Critical History. *By John J. Roche.* Springer-Verlag, New York, 1998. $79.95. xi+330 pp., hardcover. ISBN 0-387-91581-8.

The author's intention in this erudite work is to show the value of critical history in the understanding of certain key concepts of physics and engineering science such as the practical understanding of number, the use of algebra, and, more recently, dimensional explorations. It is in the pursuit of details through the copious notes and full bibliography that the working physicist or applied mathematician will find much of interest in this volume, for it is a contribution to the history of mathematics rather than a mathematical text in itself.

By "critical history" we are to understand that the historical record is to be examined in a way that attempts to clarify the meanings of concepts. Thus, even in prehistoric epochs, though necessarily in a more conjectural light than for later times, the practical concept of number can be seen in such activities as bartering or gaming, in which one group of objects had to be compared indirectly with another through the use of an intermediate group of objects, such as fingers, notches, or pebbles. This essentially nonverbal counting became impractical when large numbers were involved, and number-words developed from the names of familiar objects with obvious associations, such as "hand" with "five." The invention of recitation counting, uniting a tallying gesture with the recitation of a ladder of number-words, allowed a more powerful and convenient system of reference groups, namely, the word-gesture sequence ending in a certain word. The usefulness of this system was further increased by written numerals, which, in many cultures, appear to have grown up independently. The modern system, with a finite number of symbols whose values are determined by position, may have been invented by the Babylonians (with base 60 in mathematical and astronomical texts) but apparently reached us from Islam through Hindu numerals. Plato seems to have had both a physical interpretation of number and also the conception of number as the written numeral.

It is tempting to follow Roche down the pathways he has opened up through the forest of his prodigious reading, but a reviewer must be content to note the areas into which the author's exploration has divided his subject. From the emergence of a practical concept of number, he reviews the development of quantitative science and the use of geometry and algebra in the physics of the 14th to the 17th centuries. The exact science of the 18th century and the metric reform are treated, as well as the absolute quantification demanded by the developments in electricity and magnetism. This leaves two chapters on dimensional analysis and exploration and a few concluding remarks on the algebra of modern physics.

I wish that a more complete treatment of dimensionless numbers—both dimensionless variables and dimensionless parameters—had been given, though one cannot but sympathize with an author who is honest enough to say that the "study of dimensional analysis was, perhaps, the most exhausting labour of reading and analysis which I have ever undertaken." Perhaps part of the confusion that Roche remarks on in the present state of dimensional analysis is due to the inapplicability of the elementary form of dimensional analysis to any model but one of the type $f = Ma^x b^y c^z$. Each term in a more sophisticated model, such as a partial differential equation, may be of this form and require its own analysis, but the insight gained by making a model appropriately dimensionless (e.g., the beautiful emergence of the Reynolds number from the Navier–Stokes equations) is the chief intellectual glory of that banausic breed of natural philosopher, the engineer.

RUTHERFORD ARIS
University of Minnesota

Parametric Statistical Change Point Analysis. *By Jie Chen and A. K. Gupta.* Birkhäuser, Boston, 2000. $59.95. viii+184 pp., hardcover. ISBN 0-8176-4169-6

The formal study of change point problems dates back to the 1950s (Page (1954)). Page's main object was the detection of changes in a sequential setting and he established the statistical foundation for modern quality control. Quandt, in the late 1950s and early 1960s, addressed the question in the different framework of so-called switching or two-phase regression. Shiryayev, in a series of papers, studied change points from a Bayesian perspective in the 1960s. Since these modest beginnings, change points have permeated both the applied and theoretical literature.

A literature review up to 1998 can be found in Asgharian (1998). More recently a quasi-likelihood approach for fitting multiple change point models has been considered by Braun, Braun, and Muller (2000), while Jandhyala and Fotopoulos (1999) have treated asymptotic properties of the maximum likelihood estimator (MLE) of a change point. Kutoyants (2000) has discussed change point problems in ergodic diffusion processes. Antoniadis et al. (2000) have used wavelets for estimating the location of a change point in the hazard function when the data are subject to right censoring. Using the Bayes information criterion, Reed (2000) has studied the identification of multiple change points for a piecewise constant hazard rate.

The statistical theory of change point analysis is now well developed, and the monograph under review represents a timely account of a part of it. The book contains detailed explanation of some technical papers on parametric change point analysis. Considerable effort is devoted to presenting detailed proofs of the asymptotic distributions of likelihood procedures based on test statistics for univariate and multivariate normal distributions. The book is generally aimed at researchers and graduate students with a good background in probability and asymptotic theory.

The detailed presentation of these technical proofs should be of interest to theoretical statisticians working on different aspects of change point problems. Due to the slow rate of convergence, these results might not, however, be of importance to applied researchers. The monograph includes only a few examples, which does not seem enough to give a real sense of the wide range of important applications of the subject.

The book is organized as follows. Chapter 1 gives an overview of change point problems and the methodologies used in change point literature. Chapter 2 provides a detailed discussion of different methods, including a likelihood ratio and an informational approach, of inference about mean and variance change for the univariate normal model. The tensile strength data of Shewhart (1931) is analyzed using the SIC (Schwarz information criterion), and it is shown that the conclusion matches the one drawn by Shewhart. In Chapter 3, the results on the univariate normal given in Chapter 2 are extended to the more interesting case of the multivariate normal. Two examples presented at the end of this chapter are analyzed using an informational approach. Regression models are discussed using a Bayesian and an informational approach in Chapter 4. Enough details are provided in this chapter to make the arguments easy to follow without getting caught up in technicalities. Chapters 5 and 6 are, respectively, concerned with change points in gamma and exponential distributions. Chapter 7 contains surveys on change point problems in binomial and Poisson distributions.

In summary, the monograph under review is timely and a good starting point for both researchers and theoretically strong graduate students interested in pursuing theoretical research in nonsequential parametric single-path change point problems. It does not, however, address some important aspects of change point problems, including statistical analysis of change points in the hazard function. The sequential change point problem is almost completely left out. The book does not cover a reach class of change point problems, the so-called multipath change point problems, discussed in a series of papers by Joseph and Wolfson (see Asgharian (1998)) either. In a multipath change point problem, there are several paths, each having a possible change point. The observations may be depicted by a matrix:

$$\begin{matrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{matrix} \begin{pmatrix} X_{11} & X_{12} & \dots & X_{1m} \\ X_{21} & X_{22} & \dots & X_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ X_{n1} & X_{n2} & \dots & X_{nm} \end{pmatrix}.$$

This is a rather general setting that includes, as special cases, repeated measurements, panel data, and balanced longitudinal data, with the important twist that the time-to-effect of a stimulus might be different from one subject to another (e.g., Pauler and Laird (2000)).

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M. Asgharian
McGill University

Industrial Mathematics: Modeling in Industry, Science, and Government. *By Charles R. MacCluer.* Prentice-Hall, Upper Saddle River, NJ, 2000. $76.00. xii+308 pp., hardcover. ISBN 0-13-949199-6.

The words “industrial mathematics” mean different things to different people at different times. I thought that I knew what those words meant when I was a mathematics graduate student in the Bay Area with a part-time job at the west coast research labs for a major telecommunications firm. The group I was assigned to was heavily involved in plasma physics. Chapman and Cowling’s book *The Mathematical Theory of Near-Uniform Gases* was our bible. My job was to help the engineers and physicists in our group to understand papers on perturbation techniques for solving the Boltzmann equation and the theoretical underpinnings of the mathematical models in nonuniform gases. So it was easy for me to believe that industrial mathematics was just a branch of mathematical physics, but with a twist: the work must fit the funding (factors that affect funding are not always visible to the naked eye). My job title was listed as "engineering specialist," but I learned later that industrial concerns almost never use "mathematician" as a job title. Most mathematicians end up with job titles like "member of the technical staff."

With a brand new Ph.D. I was interviewed by the math group manager at a large, well-known think tank. He asked me what I thought industrial mathematics was. My response must have irritated him, because he loudly shot back the response "industrial mathematics is 17th-century mathematics!" Digesting this new bit of wisdom I continued my job search and eventually joined the math consulting group at a large, well-known aerospace firm. My job title was "senior engineering specialist"—clearly I was coming up in the world. Our group was heavily involved with raising communications satellites into geosynchronous orbits, and so I learned a lot about the mathematics behind celestial mechanical models. Unfortunately, I was never given much of a chance to actually work on projects; my main activity was writing proposals—because I had a Ph.D., I suppose. So, at this early stage in my career I learned another valuable lesson about mathematicians in industrial environments: you have to make yourself useful any way you can, all the while keeping an eye on deadlines and budgetary constraints. When required, the really heavy-duty mathematical modeling and analysis would very often be farmed out to consultant experts in the field (who were nearly always academics). In fairly short order I entered the ranks of academia myself, where for many years I directed a program of doing industrial mathematics (there are those words again) on campus. The program was founded at Harvey Mudd College in 1973 and is called the Mathematics Clinic; the projects are real problems that come from industry and are worked on by faculty-student teams [1]. So, my experience with industrial mathematics is rather delicately balanced between industry and academia.

Paging through Charles MacCluer's delightfully written book on industrial mathematics, the reader will be struck by the broad range of mathematical applications that one might indeed encounter in the real world. Neatly divided into 14, rather compact chapters according to the mathematical structures used, the book was designed (the author says) for senior undergraduate or master's students of mathematics, engineering, or science who are about to enter the workforce. Each chapter begins with a (very) brief presentation of the mathematical tools and results used in the interesting problems and examples covered in that chapter. A very extensive collection of exercises appears at each chapter's end; some exercises are straightforward and some require a careful reading of the problems and examples of the chapter. Other exercises are essentially challenging projects intended for a team of students, often requiring them to collect data and consult with industrial experts. Throughout the book the author time and again illustrates the value of numerical computation by presenting a MATLAB script that "solves" the stated problem. In this way the author drives home the point that although theory is important in creating mathematical models, good commercial software is almost always required in industry to provide useful results in a timely fashion.

There are a couple of aspects of industrial mathematics that definitely set it apart from applied mathematics pursued in academia. First, the mathematicians in industry almost always work together with others as members of teams (most likely, interdisciplinary teams). Second, the team must communicate effectively in written and oral reports with others in the organization who are (almost certainly) not as expert on the subject of the project as the team members are. Very often the mathematicians perform a valuable service by creating a clean and unambiguous mathematical model that captures the essence of the problem at hand and then aiding the team in the analysis of that model.

There are many books that describe mathematical models that arise in industry and then go on to show how these models can be analyzed mathematically. MacCluer's book does two things that set it apart from those books: First, MacCluer has assembled the mathematical tools that engineers often need and arranged them in 14 compactly written chapters that show how these tools are used in practice. The second feature is the final chapter, Chapter 15. It is a superbly written description of the "five common instruments of technical communication: the formal report, the memo, the progress report, the executive summary, and the problem statement." (There are also some helpful hints on preparing oral reports.) This last feature is something that engineers are usually taught by example and the occasional hastily written hand-out. I do not recall seeing these "instruments" described in this detail in any other modeling book. It is really nice to see it all together in one place so that students can refer to it when needed. I think this feature is an extremely valuable part of MacCluer's book.

This is a very ambitious book. The technical portion covers a lot of ground in 276 pages, and it is packed with lots of interesting applications, examples, exercises, and projects. In spite of the author's terse writing style he manages to present enough of the theoretical underpinnings to give advanced undergraduates a fighting chance to understand the subjects covered. It is clear, however, that an instructor using this book as a textbook for a course in industrial mathematics must be prepared to spend some considerable time with students to guide them in the unfamiliar world of constructing mathematical models and doing project work. Students would most likely also need serious help in mathematical analysis, but from my experiences students rarely need help with computerware. A word of caution to young readers, however: The topics covered in this book are by no means the only mathematics encountered in industry.

The first two chapters are a basic introduction to the kind of applied stochastic modeling that often arises in an industrial setting. Chapter 1 begins with a short primer on probability distribution functions with typical applications using the uniform, Gaussian, binomial, and Poisson distributions and ends with the not-so-typical Taguchi quality control application. Chapter 2 is a very brief introduction to the Monte Carlo method with some interesting practical applications. MacCluer uses a technique in these chapters that he repeats often throughout the book: he solves the same problems using several different approaches. In this case it is the newsboy problem: "A newsboy sells newspapers outside Grand Central Station. He has an average of 100 customers per day. He buys papers for 50 cents each and sells them for 75 cents each, but cannot return unsold newspapers for a refund. How many newspapers should he buy?" This problem is solved using a binomial distribution approach, a Poisson approach, and a Monte Carlo approach (and they all return close to the same result!). Annotated MATLAB scripts are presented to show how this optimization problem can be solved without first having an analytic solution. Among the many interesting exercises and projects at the ends of these chapters, I found several intriguingly stated ones such as, "Why do physical constants lead with the digit 1 more often than any other digit?" and "Why are the first several pages of a logarithm table more worn than the last several?" (Do students today even know what a log table is?)

The pace of the next two chapters picks up considerably. Chapter 3 deals with the manipulation of discrete data and the processing of discrete signals. The main tool, the z-transform, is introduced at the outset and its properties are quickly summarized and then applied to the solution of linear recursions and the design of digital filters. Stability of a filter is discussed briefly and Bode plots are brought up in this connection. The chapter ends with a quick introduction to closed-loop feedback systems and the interesting application of designing an automobile digital cruise control system. A delightful feature of MacCluer's writing style is his insightful way of explaining things in a conversational tone. A fine example of this occurs at the end of this chapter in a section titled "Why Decibels?" Chapter 4 covers the spectral analysis of on-line signals using the discrete Fourier transform (DFT) to process a data stream generated by samples of the signal at equally spaced time intervals. The basic properties of the DFT are quickly summarized and then applied to filter design. This is followed up by a discussion of the fast Fourier transform (FFT) as a way to speed up DFT calculations and then applied to the question, "What makes the oboe so distinctive an instrument?" The chapter ends with an application of the DFT to image processing. With all the ground covered, it is hard to believe that Chapter 3 and Chapter 4 are only 22 pages and 14 pages long, respectively. This is a masterful tour de force by MacCluer in that everything needed appears there—with not a wasted word.

The rather short next four chapters, Chapters 5–8, continue with other topics that are useful for modeling in a discrete environment. The nine-page Chapter 5 covers the simplex algorithm in linear programming remarkably well. The short Chapter 6 neatly covers the basics of regression for discrete as well as continuous data, the latter after a quick review of Hilbert spaces. Chapter 7 is a three-page introduction to cost-benefit analysis with many interesting project exercises at the chapter's end (e.g., "Should class size be reduced nationwide in the public schools?"). Chapter 8 is a ten-page introduction to modeling in microeconomics, covering the basics such as supply, demand, revenue, cost, profit, elasticity of demand, competition, and production, and ends with the Leontiev input/output model. MacCluer has done a superb job in presenting the material in these four chapters using so few pages. For those who want more, at each chapter's end MacCluer includes an annotated list of references, which he has clearly thought out carefully.

The next three chapters, Chapters 9, 10, and 11, are devoted to differential equations and together comprise about one-third of the book. Chapter 9 gives a very basic introduction to ordinary differential equations (ODEs), mostly via models in mechanics. Solution techniques for undriven linear ODEs and systems with constant coefficients are presented in a very concise fashion. The chapter ends with a problem about traffic flow on a congested expressway; this problem is modeled by a system of differential-delay equations that require the frequency-domain methods of the next chapter to solve—a very nice segue to Chapter 10. In this 35-page chapter MacCluer develops the Heaviside operational calculus to solve ODEs (and delay-ODEs) in the frequency domain. Along the way the notion of stability is also characterized in the frequency domain, as are Bode plots and Nyquist analysis. These concepts are used in filter design, in using feedback to stabilize plants, and in controlling servo mechanisms. Near the chapter's end MacCluer returns to the problem of designing an automobile cruise control system (generalizing the digital version of this problem given in section 3.7). To my mind this is the toughest chapter in the book, but it is jam-packed with good stuff. Chapter 11 covers basic solution techniques for the standard partial differential equations, which arise in applications, primarily the method of separation of variables and related techniques. Along the way a number of interesting applications are touched upon: heat conduction, the plucked string, the motion of a circular drum head, steady-state problems, a quantum particle, the Helmholtz phasor method for finding periodic steady states (too bad MacCluer didn't apply this method to find the optimal depth for a wine cellar!), and others. Near the chapter's end the author does a traffic flow problem and introduces characteristic curves. This is another fully packed chapter.

The final three technical chapters, Chapters 12–14, deal with approximation techniques. Chapter 12 discusses Euler, Heun, and Runge–Kutta fourth-order methods for solving ODEs and presents MATLAB scripts for carrying out the computations. I always thought that these methods were of first, second, and fourth order, respectively, but MacCluer lists them as second-, third-, and fifth-order methods, respectively. He references Gilbert Strang's book *Introduction to Applied Mathematics* for that bit of information, but I could not find it in my 1986 edition (I don't have the 1961 edition that MacCluer listed in his bibliography). Another small point: the MATLAB routine ODE45 is not the RK4 algorithm MacCluer describes in (12.18a) and (12.18b). This chapter also presents Euler and Crank–Nicolson methods for the one-dimensional heat equation. The delightfully written Chapter 13 presents Galerkin's method for finding approximate solutions of boundary/initial value problems for differential equations and eigenvalues for differential operators. A brief introduction to finite element methods is also included. Again MATLAB scripts are presented that are very useful in the exercises. Chapter 14 is a brief introduction to splines.

MacCluer is in good company when he encourages his students to follow his course on industrial mathematics with project work on a real problem from industry. Ever since the mid-1960s, when computers began to become generally available to undergraduates, the call has gone out to find ways to introduce applied mathematics and "hands-on" mathematical modeling into the undergraduate curriculum, topped-off with an opportunity for students to work on "real" problems from industry. The question has always been how this could be done in a meaningful and effective way. NSF, SIAM, the MAA, and others have devoted a great deal of time and talent to this question since the mid-1960s, and a great deal of progress has been made. The contributors to this quest are far too numerous to list here, but it is worth mentioning a few unusual ones in passing. In 1969 two engineers at Widener University published the *high-school* case studies text *You and Technology*, designed to introduce students early to "hands-on" project work [2]. In 1976 COMAP (Consortium for Mathematics and Its Applications) was established to support the introduction of applied mathematics materials into the high-school and undergraduate mathematics curricula [3]. A few years later COMAP took over the administration of the international competition in mathematical modeling, called the MCM (Mathematical Competition in Modeling) [4]. The MCM invites teams of undergraduates to work for 72 hours on one of three proposed industrial problems and ends with a report summarizing the team's work on the modeling and analysis of their selected problem. The fall semester 2000 saw the inauguration of IPAM (Institute of Pure and Applied Mathematics), which joins two other NSF-funded mathematics research institutes. The principal objective of IPAM is to encourage cross-fertilization between the mathematical sciences and other scientific disciplines [5]. Beginning in 2001, IPAM will have a summer math clinic program where undergraduates will conduct research on projects arising from industrial sponsors.

In conclusion, I would like to highly commend Charles MacCluer for taking on the enormous task of pulling together a compendium of mathematical topics that are useful in industry and to write about them clearly, concisely, and coherently. It must have been difficult to try to design the chapters to be independent, and in this he mostly succeeds. I love his oft-times conversational tone in which he gives his personal insight into the material covered. This book should be in the library of anyone who is even remotely contemplating becoming involved with industrial mathematics.

REFERENCES

[1] Harvey Mudd College Mathematics Clinic, founded 1973, Director: Michael Raugh. The web site http://www.math.hmc.edu/clinic gives a detailed description of the clinic program.

[2] *You and Technology*, a high-school case studies text with Teacher's Guide, edited by N. Damaskos and M. Smyth, Widener University, Chester, PA, 1969.

[3] COMAP (Consortium for Mathematics and Its Applications), Lexington, MA, founded in 1976. Executive Director: Solomon Garfunkel. Visit the web site http://www.comap.com for a complete listing of all COMAP publications and activities.

[4] MCM (Mathematics Competition in Modeling). Founded in 1984 by Ben Fusaro, currently administered by COMAP and directed by Frank Giordano. The web site http://www.comap.com/undergraduate/contests/mcm/index.html gives more details. The MCM was designed to be the applied mathematics analog of the Putnam exam.

[5] IPAM (Institute of Pure and Applied Mathematics) was founded in 1999 as the most recent NSF-funded mathematics reseach institute. Headquartered at UCLA, the current Director is Tony Chan. Beginning in 2001,

IPAM will feature a summer program of real-world industrial projects for teams of faculty and undergraduates. More detail is at the web site http://www.ipam.ucla.edu.

Robert Borrelli
Harvey Mudd College

The Development of Prime Number Theory. *By W. Narkiewicz.* Springer-Verlag, New York, 2000. $94.00. xii+448 pp., hardcover. ISBN 3-540-66289-8.

The prime numbers, $2, 3, 5, 7, 11, \ldots$, have long fascinated people. Questions about primes are generally easy to understand, but the answers can be very hard to find. For example, given that primes seem to occur so erratically, it is possible to say anything about how they are distributed? Practical questions for subjects like cryptography include, How does one determine whether a large number is prime? and How does one factor a number if it is not prime? The theme of Narkiewicz's book is the development of prime number theory. At the cost of giving away the plot, we begin this review by describing the historical milieu of our subject.

The infinitude of the primes was established in Euclid's *Elements* some 2300 years ago. In the 1830s, P. G. L. Dirichlet introduced a remarkable set of new ideas involving arithmetic functions and an associated class of infinite series that we today call Dirichlet characters and Dirichlet L functions, respectively, to show that each arithmetic progression $\{a \bmod m\}$, with a and m relatively prime, contains an infinite number of primes. The next question—Does an irreducible quadratic polynomial $P(x)$ with integer cofficients necessarily represent primes for an infinite number of positive integer arguments?—remains unsettled to this day.

Another reasonable question is, What is the size of the nth prime? No progress was made on the question in this form. Around 1800 C. F. Gauss and A. M. Legendre had the insight to consider the inverse question about the size of $\pi(x)$, the number of primes not exceeding x. Independently, they conjectured the prime number theorem (PNT)

$$\lim_{x\to\infty} \frac{\pi(x)}{x/\log x} = 1.$$

Actually, Gauss thought to compare $\pi(x)$ with

$$\operatorname{li} x := \lim_{\epsilon\to 0+} \left(\int_0^{1-\epsilon} + \int_{1+\epsilon}^{x} \right) \frac{dt}{\log t},$$

a function that turns out to be significantly nearer to $\pi(x)$ than $x/\log x$ is.

Around the middle of the 19th century two ingenious methods were put forth to estimate $\pi(x)$. P. L. Chebyshev discovered a combinatorial inequality which, when combined with elementary real variable techniques, established for the first time the true order of magnitude of $\pi(x)$. But Chebyshev's ideas did not admit easy extension, and no progress was made toward an "elementary" proof of the PNT for the next 100 years. G. F. B. Riemann proposed a formula for calculating $\pi(x)$ using an integral in the complex domain of a function that is today named for him. The Riemann zeta function is defined on $\{s \in \mathbf{C} : \Re s > 1\}$ by

$$\zeta(s) = 1 + 2^{-s} + 3^{-s} + \cdots$$

and is extended to $\mathbf{C}$ as a meromorphic function. Riemann also put forth several assertions about the zeta function, all of which, save one, were established in the next 50 years. The remaining conjecture, the famous Riemann hypothesis that the real part of each nonreal zero of $\zeta(s)$ is $1/2$, remains unproved to this day.

J. Hadamard took up Riemann's approach to estimate $\pi(x)$ and developed important parts of complex function theory for this purpose. In 1896 he and C. J. de la Vallée Poussin independently proved the PNT, one of the great achievements of 19th century mathematics. About a hundred years after the work of Chebyshev, A. Selberg found a relation with which he and, independently, P. Erdös obtained an "elementary," i.e., real variable, proof of the PNT.

There are now a number of notable books on prime number theory. E. Landau's *Handbuch*, first published in 1909 and reprinted with an update by P. T. Bateman in 1953, is distinguished in that it treated nearly everything that was then known about the distribution of primes and served to bring prime number theory into the mainstream of mathematics. P. Ribenboim's *Prime Number Records* is probably the all-time best-selling book on prime numbers. Some other very accessible books on the subject are those of Ingham, Ellison and Mendes-France, and Tenenbaum and Mendes-France.

The book under review is a work of great scholarship on which the author has lavished loving attention. Narkiewicz speaks most of the languages of his original sources, and he has provided numerous insightful quotes from the masters of his subject, along with English translations. The bibliography itself is a treasure, providing 78 pages (!) of important sources. Most mathematical results discussed are proved fully, but some technical matters are discussed without proof. There are instances where arguments could be simplified (e.g., on p. 54, where a gamma function trick of Euler could be replaced by integration by parts), but the techniques the author develops are all valuable.

Narkiewicz says in the preface, "This is not a historical book since we refrain from giving biographical details...." Nevertheless, the book provides an excellent account of the way in which the theory of the distribution of prime numbers developed through "the end of the first decade of the 20th century." Many recent achievements are also discussed, including a 1998 reference, in the chapter titled "Early Times," but the greatest detail is devoted to the earlier work.

Principal topics of the book include prime number formulas and criteria; the theorem of Dirichlet on the infinitude of primes in arithmetic progressions, a topic for which the author clearly has special enthusiasm; Chebyshev's theorem on the order of magnitude of the prime counting function $\pi(x)$; properties of the Riemann zeta function; the PNT and missteps on the way to its proof; and subsequent work on such topics as the location of zeros of $\zeta(s)$, changes of sign of $\pi(x) - \operatorname{li} x$ (see below), and results and conjectures of G. H. Hardy and J. E. Littlewood.

Some significant topics of more recent times that are not treated by Narkiewicz include the large sieve, which serves as a more-than-adequate substitute for lack of knowledge of the location of zeros of L functions in many problems concerning primes in arithmetic progressions; trigonometrical sum methods, which lead to improved zero-free regions for the Rieman zeta function, which in turn yield better error estimates of $\pi(x) - \operatorname{li} x$; and automorphic form methods, which play roles in sieve methods and locating zeros of L functions.

The book contains a collection of interesting problems at the end of each chapter. As a sample, here is problem 1 from Chapter 1: (Hacks 1893) Prove that $n \in \mathbf{Z}$ is prime if and only if

$$\sum_{a=1}^{n-1} \sum_{b=1}^{n-1} \left[\frac{ab}{n}\right] = \frac{(n-1)^2(n-2)}{4}.$$

The connection of the sum with prime numbers becomes clearer when one considers the residues of ab modulo n. Another example, problem 1 of Chapter 3, which stems from Chebyshev, concerns the density of the primes: Let $\{a_n\}$ be a sequence of positive real numbers. Show that

$$\sum_{n=2}^{\infty} \frac{a_n}{\log n} < \infty \iff \sum_p a_p < \infty.$$

An additional condition, such as monotonicity of $\{a_n\}$, is needed for this result to be valid.

One of the charming episodes in the history of prime number theory treated by Narkiewicz is the determination of the relative sizes of $\pi(x)$ and $\operatorname{li} x$. Tables of primes suggested that the inequality $\pi(x) < \operatorname{li} x$ holds for all $x > x_0 = 1.451369\ldots$, the point where $\operatorname{li} x$ turns positive. (The inequality between $\pi(x)$ and $\operatorname{li} x$ is unfortunately stated backwards on p. 322.) Riemann gave a theoretical basis for this observation by commenting in his famous memoir that the analytic method he proposed actually applied to the weighted prime-power counting expression

$$\Pi(x) := \pi(x) + \frac{1}{2}\pi(x^{1/2}) + \frac{1}{3}\pi(x^{1/3}) + \cdots,$$

which is larger than $\pi(x)$ by about $\sqrt{x}/\log x$. It came as a complete surprise when, in 1914, J. E. Littlewood proved that the expression $\pi(x) - \operatorname{li} x$ changes sign on an infinite sequence of numbers x tending to infinity. The first numerical estimate of a sign change beyond x_0, by S. Skewes, was the prodigious number $10^{10^{10^{100}}}$. It is now known, through the work of H. te Riele, that a change of sign occurs before $7 \cdot 10^{360}$.

In summary, this is a delightful book to read and will be well received. It contains much material for either a basic course on the distribution of prime numbers or a mathematical history course on the subject. The bibliography itself will make this book attractive to practicing number theorists. Professor Narkiewicz is to be congratulated warmly on his contribution to the literature of prime number theory.

HAROLD G. DIAMOND
University of Illinois, Urbana-Champaign

Measuring Computer Performance: A Practitioner's Guide. *By David J. Lilja.* Cambridge University Press, Cambridge, UK, 2000. $39.95. xv+261 pp., hardcover. ISBN 0-521-64105-5.

Measuring computer performance has always been more of an art than a science. Lilja's book attempts to put more science into the process. *Measuring Computer Performance* sets out the fundamental techniques used in analyzing and understanding the performance of computer systems. Throughout the book the emphasis is on practical methods of measurement, simulation, and analytical modeling.

The author discusses performance metrics and provides detailed coverage of the strategies used in benchmarking programs. He gives intuitive explanations of the key statistical tools needed to interpret measured performance data. He also describes the general techniques used in the design of experiments and shows how the maximum amount of information can be obtained for the minimum effort.

The first chapter begins with an introduction to the basic ideas of measurement, simulation, and analytical modeling. It describes some of the common goals of computer systems performance analysis. The problem of choosing an appropriate metric of performance is discussed in Chapter 2, along with some basic definitions of speedup and relative change.

The next three chapters provide an intuitive development of several important statistical tools and techniques. Chapter 3 presents standard methods for quantifying average performance and variability. It also introduces the controversy surrounding the problem of deciding which of several definitions of the mean value is most appropriate for summarizing a set of measured values. The model of measurement errors developed in Chapter 4 is used to motivate the need for statistical confidence intervals. The ideas of accuracy, precision, and resolution of measurement tools are also presented in this chapter. Techniques for comparing various system alternatives in a statistically valid way are described in Chapter 5. This presentation includes an introduction to the analysis of variance, which is one of the fundamental statistical analysis techniques used in subsequent chapters.

While Chapters 3–5 focus on the use of interpretation of measured data, the next two chapters emphasize tools and techniques for actually obtaining these quantities. Chapter 6 begins with a discussion of the concepts of events. It also describes several different types of measurement tools and techniques, including interval timers, basic block counting, execution-time sampling, and indirect measurement. The underlying ideas behind the development of benchmark programs are presented in Chapter 7, along with a brief description of several standard benchmark suites.

Chapter 8 uses a discussion of linear regression modeling to introduce the idea of developing a mathematical model of a system from measured data. Chapter 9 presents techniques for designing experiments to maximize the amount of information obtained while minimizing the amount of effort required to obtain this information. The fundamental problems involved in simulating systems are discussed in Chapter 10. Finally, Chapter 11 concludes the text with a presentation of the fundamental analysis modeling techniques derived from queuing theory.

In addition, a glossary of some of the more important terms used in the text is presented in Appendix A. Several common probability distributions that are frequently used in simulation modeling are described in Appendix B. Appendix C tabulates critical values used in many statistical tests described in the earlier chapter.

The book can be used as a primary text in a one-semester course for advanced undergraduate and beginning graduate students in computer science and engineering who need to understand how to rigorously measure the performance of computer systems. The book is very informative, with a rich collection of examples, exercises, and references. As such, this book would be an important addition to the collection of anyone interested in measuring computer performance.

JACK DONGARRA
University of Tennessee

Self-Organization and the City. *By Juval Portugali.* Springer-Verlag, New York, 1999. $99.00. xvii+352 pp., hardcover. ISBN 3-540-65483-6.

As it says on the back cover, this book combines the theory of complex self-organizing systems with the social theory of cities and urbanism. This combination of natural sciences and humanities is not so frequently met. The synergetical approach as developed by Haken in the 1970s has facilitated the extended use of concepts and methods of the exact sciences to complex dynamical structures in the life sciences. Mathematicians have their theories of bifurcation and chaos to explain sudden change in systems. In physics comparable changes are seen as phase transitions and critical behavior is captured in prototype systems such as the sandpile model. In the life sciences there are phenomena that can be better understood by analogy with physical systems: self-organization is a prominent case. From this book and other studies one can become convinced of the general validity of principles about the way that "players'" interaction may lead to a spatiotemporal structure. In the book nice examples of applying cellular automata to city development are given. This makes it unnecessary to come up with Benard's experiment on spatial structures in convective flow to explain hexagonal compartments in ideal cities. Self-organization in slime mold populations and plasticity in neural processes would have provided a more inspiring analogy.

The reader should not expect practical tools for city planning from this book: it is a completely worked-out case study of self-organization using synergetics. The ideas behind the part on "Planning in a Self-Organizing City" are not so clear: Why does self-organization need planning? Some sociological theories of the city, such as Weber's theory of the ideal city and Wittgenstein's network theory, could also have been left out, as self-organization plays no role in them. However, the connection Alexander makes between a physical object (from the city of Jericho to Chicago) that changes in time and the notion one has of it has interesting mathematical potential in, e.g., set theory and continuation theory. From the other side this concept can be linked to Mead's symbolic interactionism in sociology.

At its start in the 18th century, sociology was strongly influenced by natural science with its successes (positivism). However, sociology quickly went its own way as this approach was unrewarding at that time. Maybe the two are merging again. We see it already in Hagerstrand's theory of the spread of innovations, which comes very close to Fisher's model of the spread of a genotype in a population: both belong to the more general class of reaction diffusion systems. This book makes a strong case for a next step guided by the theory of synergetics.

JOHAN GRASMAN
Wageningen University

Regular Variation and Differential Equations. *By Vojislav Marić.* Springer-Verlag, New York, 2000. $29.80. x+127 pp., softcover. ISBN 3-540-67160-9.

A positive measurable function defined on a half-line is called regularly varying (at infinity) of index α if, for some $\alpha \in R$ and each $\lambda > 0$, $\lim_{x\to\infty} \frac{\rho(\lambda,x)}{\rho(x)} = \lambda^\alpha$. If $\alpha = 0$, ρ is called slowly varying. If

$$\lim_{x\to\infty} \frac{\rho(\lambda x)}{\rho(x)} = \begin{cases} \infty & \text{if } \lambda > 1 \\ 0 & \text{if } 0 < \lambda < 1 \end{cases},$$

ρ is called rapidly varying of index ∞, and if

$$\lim_{x\to\infty} \frac{\rho(\lambda x)}{\rho(x)} = \begin{cases} 0 & \text{if } \lambda > 1 \\ \infty & \text{if } 0 < \lambda < 1 \end{cases},$$

ρ is called rapidly varying of index $-\infty$.

In 1930, J. Karamata introduced the class of regularly varying functions in connection with his study of Tauberian theorems. A number of related classes that play analogous roles were developed subsequently. The Karamata classes have proved effective in treating a variety of problems in complex analysis and probability theory as well as harmonic analysis.

The present work provides applications of the Karamata theory to study the behavior of solutions of various second-order differential equations; a special third-order equation arising in boundary-layer theory is also treated. A substantial portion of the book is based on the extensive work of the author and his coworkers, especially M. Tomić.

For the linear equation

$$y'' + f(y) = 0, \tag{1}$$

integral conditions are used to determine the Karamata variation class and, in many cases, to provide asymptotic estimates of solutions. While much of the material is concerned with the case where $f < 0$, some results are obtained providing asymptotics of nonoscillatory solutions in cases where f has arbitrary sign.

Several additional classes of functions related to the Karamata classes are also introduced, including the more general set of regularly bounded functions, the de Haan class, and the Beurling slowly varying functions. These have been used to provide alternate approaches in the nonoscillatory case, and in addition yield results on the behavior of zeros of oscillatory solutions of equation (1).

The latter part of the book provides estimates on solutions of certain nonlinear equations, including a generalized Fermi–Thomas equation and the third-order McLeod equation $y''' - yy'' + \lambda(1 + (y')^2) = 0$. The book concludes with a brief appendix summarizing salient properties of the Karamata functions.

An excellent treatment of the Karamata theory and related material, together with several extensive applications, is given in the treatise of N. H. Bingham, C. M. Goldie, and J. L. Teugels, *Regular Variation* (Cambridge University Press, 1987), but only passing mention is made there of applications to differential equations (many of these have been developed since 1987).

The present work is a very readable and valuable adjunct to the book of Bingham, Goldie, and Teugels and provides a useful introduction to the Karamata theory for students as well as specialists in differential equations.

LOUIS J. GRIMM
University of Missouri-Rolla

Visual Quantum Mechanics, Selected Topics with Computer-Generated Animations of Quantum-Mechanical Phenomena. *By Bernd Thaller.* Springer-Verlag, New York, Berlin, Heidelberg, 2000. $69.95. ix+283 pp., hardcover. ISBN 0-387-98929-3.

The new introductory textbook by Bernd Thaller, *Visual Quantum Mechanics*, has two distinctive features: a well-produced CD-ROM and an emphasis on dynamics. These two features work together well and make for a remarkable publication, which mathematically oriented readers will appreciate for the care it takes with rigorous aspects of physics.

Thaller's is not the first introduction to quantum physics that emphasizes dynamics, but there have not been many; the 1965 undergraduate textbook by Feynman and Hibbs [FH65] has been virtually the only one. Almost every other textbook up to the present day begins by developing the theory of bound states, which are eigenfunctions of the Schrödinger equation, and only later discusses a single time-dependent phenomenon, scattering. Usually, even scattering is presented in a time-independent formulation, achieved with the

Fourier transform. This strong preference for time-independent problems is a strange feature of quantum pedagogy, but it has little to do with the strangeness of quantum mechanics itself. It is true that the important topic of energy levels is stationary in time, but this is a secondary reason. The main reason for the static bias is, or was, practicality. Virtually all the illustrative problems in quantum mechanics that can be dissected in front of an undergraduate student are solved by separating the space and time variables, so in effect they become static. The standard illustrative models are the particle in a box, the free particle (plane waves), the harmonic oscillator, and, fortunately for Schrödinger in 1926, the hydrogen atom. Other models are traditionally treated by perturbation theory, which in its many variants becomes a major part of the quantum mechanician's education.

In their idiosyncratic book Feynman and Hibbs formulated the subject dynamically from the start. The early chapter, "Developing the Concepts with Specific Examples," provided some models on which the student's time-dependent intuition can be founded. Among these are the propagators (fundamental solutions) for the Schrödinger equation with electric and magnetic fields. Still, there are only a handful of really accessible problems, necessarily highly symmetric ones. While Feynman and Hibbs's book is an excellent guide to understanding path integrals as physicists do (no pretense of mathematical rigor is made), the fully time-dependent approach to introductory quantum mechanics has remained out of the mainstream. In 1965, time-dependent examples were both few in number and lacking in immediacy.

That was then. Today, with numerical computations and multimedia software, it is possible to write a textbook that shows movies of time-dependent wave functions. Thaller's *Visual Quantum Mechanics* is the first to seriously attempt this, by including a CD-ROM with a large number of dynamic illustrations of quantum phenomena.

In this, still early stage of the information revolution, one approaches publishing innovations with trepidation. Will the multimedia in this one fail to measure up to its hype? Will that one require me to purchase ever more software packages? Will the operating system hang, blaming me for illegal operations? None of these fears materializes with Thaller's book. The multimedia supplements are impressive, work flawlessly, and contribute materially and essentially to the educational experience. The successful experiment in multimedia publishing convinces this reviewer that the day is not long distant when every successful undergraduate science text will, in one way or other, be a dynamic publication.

As its subtitle indicates, the book presents "selected topics" in quantum mechanics. It could profitably be assigned to American undergraduates even without the CD-ROM, as a supplementary source for material not found in the typical textbook, such as "Schrödinger cat states," or for greater mathematical depth when discussing topics like electromagnetic interactions, a research specialty of Thaller's. The level of the book is introductory, requiring only basic physics and calculus courses. It is thus more in competition with texts such as [G95] or [L97], than with [LL97] or [M00]. *Visual Quantum Mechanics* is particularly appealing for mathematics students, as it is much more accurate when discussing mathematical ideas than any other introductory text of which I am aware. It would be excellent preparation for someone who wishes later to study mathematical quantum mechanics using [BEH94] or [T81]. This is managed without overwhelming the physics student—or instructor—with mathematical detail. The writing style is informal and engaging, and the English is completely fluent, although a few harmless lapses reveal that it is not the author's mother tongue.

It is the CD-ROM that makes the book. The solutions of Schrödinger's equation are complex-valued functions of space and time, so the first issue that Thaller had to confront was how to present them visually. The solution was to encode complex phases with a color wheel, while amplitude is represented by intensity. Quicktime movies then allow one to view the time-evolution of wave functions in two space dimensions. I tested the software out first on a Macintosh, since it was created on that platform and Quicktime is an Apple product, and then on a portable Dell PC running Windows 98. The software looked and felt the same on both platforms. It started intuitively, and in both cases prompted me for a painless and quick upgrade/installation of Quicktime over the internet. It then ran without a hitch. The software does not operate on Unix or Linux platforms, although a determined devotee of these systems could probably succeed in viewing the movies. The underlying calculations were done with Mathematica, but this is invisible to the user who wishes merely to watch quantum mechanics in motion. For those who wish to tinker, they can find the Mathematica code on the CD-ROM or link to it directly from the multimedia presentations.

Are there really so many ideas that are better conveyed with multimedia than with a static textbook? Thaller has provided 137 modules illustrating different topics, ranging from old standards like the double-slit experiment to such modern topics as squeezed states, quantum waveguides, and tunneling microscopy. Even the old standards are enhanced; in the module on the double-slit experiment, for example, the reader can use a slider to adjust the spacing of the slits and see the effect. Seeing interference patterns and other effects in the animations immediately engages the reader's curiosity. I would venture to say that not only the beginning student, but even the seasoned scientist may benefit, by getting a clue to a new phenomenon or theorem.

Some of the modules illustrate mathematical topics with other uses besides quantum mechanics. These range from analytic functions to Fourier transforms to generalized derivatives. Even Sobolev and Hardy inequalities make an appearance. Some mathematical topics that are absent or merely mystifying in many quantum-mechanics texts come alive. Notable among these are the role of boundary conditions and the notion of the domain of definition of an operator. In one of the modules, subtitled "Strange Behavior of the Unit Function," we watch the time-evolution of the constant function in a box, that is, the time-evolution generated by the Dirichlet Laplacian for a function that is not in its domain of definition. It is a theorem that the unitary group generated by the Dirichlet Laplacian can evolve any square-integrable function, but it is still entertaining to see it struggle.

Some stills from Thaller's movies are available for your viewing at the book's web site:

http://www.kfunigraz.ac.at/imawww/vqm/.

It is reasonable to ask whether a book of "selected topics" is broad enough to be the principal text in a course on quantum mechanics, as an alternative to the many books already on the market. Indeed, many who teach in physics departments will not be willing, for now, to adopt it. They will feel that too many topics in the canon are discussed inadequately or neglected entirely. Among these are spin and statistics; angular momentum; atomic, molecular, and nuclear structure; many-body problems; perturbation theory; group representations; and path integrals. The good news is that most of these topics are planned for a second volume, which is currently being written.

In sum, *Visual Quantum Mechanics* can be profitably used as a major supplement in an undergraduate course on quantum mechanics, and it will certainly enliven the curriculum. It could be a superb stand-alone text in a short physics course or a seminar for mathematically oriented students. When the second volume appears, probably in about two years, the text should be hefty enough to justify being adopted as the principal textbook in an in-depth physics course and may have a major impact on the curriculum of the future.

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EVANS M. HARRELL
Georgia Institute of Technology

Modeling and Computation of Boundary-Layer Flows. *By Tuncer Cebeci and Jean Cousteix*. Springer-Verlag, New York, 1999. $95.00. xv+469 pp., hardcover. ISBN 3-540-65010-5.

The book of Cebeci and Cousteix deals with the forgotten art of computing boundary layer flows, both laminar and turbulent. It is mainly a compilation of work published in the 1970s and earlier (2/3 of the references). Thus the methodology discussed mostly predates the numerical solution of the Reynolds averaged Navier–Stokes equations, which today completely dominates engineering CFD (computational fluid dynamics). It is quite informative to learn in Chapter 1 that three current important aeronautical engineering problems can be solved without resorting to the comparatively computationally expensive Navier–Stokes equations, something I am sure has been forgotten by many of the new generation of CFD users.

The positive aspect of summarizing important 20- to 30-year-old engineering computational methodology also points to the negative aspects of the book: the material is old and few new updates are included. One example is the discussion of the effects of turbulence outside the boundary layer on the laminar-turbulent transition process inside the boundary layer. This is an area where seminal work has been published during the 1990s and whose main references in the book are from 1938, 1948, 1955, and 1967.

Traditional boundary layer theory, including computer programs (available from the authors on request), are covered in Chapters 2–5. In addition, numerous valuable exercises and even computer codes for the calculation of the outer inviscid flow (panel methods) are included.

Transition prediction is covered in Chapters 6, 7, and 11 in a careful but rather traditional manner, where new methodology such as that based on the so-called parabolic stability equations (PSEs) is not described. In Chapter 8, turbulence and turbulence modeling are covered in a rather sketchy way.

The most interesting material, at least from an applications point of view, is found in Chapters 9, 10, and 12, where three-dimensional boundary layers and boundary layer separation are included. These are areas where boundary layer computations may still very well compete with full solutions of the Reynolds averaged Navier–Stokes equations in real aeronautical applications. In fact, boundary layer methods are likely to be both more accurate and less computationally expensive. However, it is not clear from reading the book what the limitations or difficulties are when these methods are applied in a real flow situation, in particular regarding the interactive boundary layer theory applied to flow separation. Flow separation is one of the most difficult phenomena to predict with any computational method and is something for which Navier–Stokes methods largely fail. If boundary layer methodology can be extended and applied in a sucessful manner, this is worth a second look for the aeronautical engineer.

DAN HENNINGSON
Royal Institute of Technology, Stockholm

Constructive Methods for Linear and Nonlinear Boundary Value Problems for Analytic Functions, Theory and Applications. *By V. V. Mityushev and S. V. Rogosin*. Chapman & Hall/CRC Press, Boca Raton, FL, 2000. $84.95. viii+283 pp., hardcover. ISBN 1-58488-057-0.

Many results of the Russian mathematical school have been presented in the form of so-called *closed form solutions*. This was the strategy of the classic books by F. D. Gakhov [1] and N. I. Muskhelishvili [2]. Petr E. Tovstik, Head of the Department of Theoretical and Applied Mechanics at St. Petersburg University, Russia, wrote in the foreword to *Asymptotic Methods in Mechanics* [3] that, with the advent of powerful computers and well-developed numerical methods, analytical solutions have been unjustly underestimated. Other examples of the Russian and Eastern European strategy using analytical methods are found in [4] and [5] as complements to numerical methods.

The book under review is self-contained and has many of its roots in the Russian mathematical school. The authors describe a number of constructive methods for boundary value problems, mainly for entire functions.

Boundary value problems go back to Riemann, Hilbert, and Poincaré. Such linear and nonlinear problems are found in elasticity, hydrodynamics, composite materials, diffraction, electrodynamics, queueing theory, and so on. The linear theory for mechanical and physical models is highly developed. On the other hand, the theory of nonlinear boundary value problems is not yet well unified and is still incomplete. Meanwhile, there exists great interest in more complicated nonlinear mechanical and physical problems that cannot always be solved by linear methods.

Most of the linear problems treated in this book generalize the following two problems. The first one is the $\mathbb{C}$-linear conjugation problem: find two functions $\Phi^+(z)$ and $\Phi^-(z)$, which are analytic in the interior and exterior of a certain Jordan curve L, respectively, such that

$$(1) \quad \Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in L,$$

where G and g are given complex-valued functions. The second one is the Riemann–Hilbert problem: find a function $\Phi(z)$ which is analytic in the interior or the exterior of L such that

$$(2) \quad \operatorname{Re} \overline{[a(t) + ib(t)]\Phi(t)} = c(t), \quad t \in L,$$

where a, b, and c are given real-valued functions.

In this book, the authors present a systematic discussion of (2) for multiply connected domains and several types of coefficients a, b, and c.

The nonlinear problems studied in this book include the following.

• General nonlinear conjugation problems of power type:

$$(3) \quad [\Phi^+(t)]^{\alpha^+(t)} = G(t)[\Phi^-(t)]^{\alpha^-(t)} + g(t), \quad t \in L,$$

where α^+ and α^- are analytic in the interior and exterior of L, respectively.

• Nonlinear conjugation problems of multiplication type:

$$(4) \quad \Phi^+(t)\Phi^-(t) = G(t), \quad t \in L.$$

• The general Riemann–Hilbert problem of power type:

$$(5) \quad \operatorname{Re} [\overline{A(t)}\Phi^\alpha(t)] = c(t), \quad t \in \mathbb{T},$$

where $\mathbb{T}$ denotes the unit circle.

• Linear fractional conjugation problems:

$$(6) \quad \Phi^+(t) - A(t)\Phi^-(t) - B(t)\Phi^+(t)\Phi^-(t) = G(t), \quad t \in L,$$

and so on.

For doubly connected circular domains, the authors use a method of fractional equations to obtain some nice results. An interesting one is the complete solution of the Riemann–Hilbert problem for multiply connected domains.

One valuable aspect of the book is the inclusion of many results from Russian journals and 295 bibliographical entries that are not always easily accessible by non-Russian mathematicians.

This book is useful for experts in analytic function theory and other specialists, in particular, scientists and engineers who want to complement numerical methods with analytical results.

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Xinhou Hua and Rémi Vaillancourt
University of Ottawa

Elementary Lectures in Statistical Mechanics. *By George D. J. Phillies.* Springer-Verlag, New York, 1999. $69.95. xvi+430 pp., hardcover. ISBN 0-387-98918-8.

This book is intended for advanced undergraduates in physics and/or beginning graduate students in physics and physical chemistry. It covers the basics of statistical mechanics, concluding with some advanced topics related to the author's research interests. It is organized into five main parts, with each part subdivided into several "lectures." A lecture is thought to be covered in 1.5 to 2 hours of class time. The five parts contain 9, 5, 6, 7, and 3 lectures, respectively. Thus the lectures total 30 and might be covered in 45–60 hours of class time. Throughout the text there are a total of seven "asides." These are about the length of a lecture and provide supplementary material and/or ease the transition between successive lectures. Each lecture is followed by homework problems and a list of references. All the problems have "qualifiers," and these briefly summarize the problem's content and whether the use of a computer is required.

The author follows Gibbs in preferring the canonical ensemble to the microcanonical one. The first main part is called "Fundamentals: Separable Classical Systems." After a brief historical introduction the author discusses averaging and basic statistical ideas; time and ensemble averages are compared. Then the canonical ensemble and canonical partition functions are introduced. This concludes the first three lectures; these are thought to form the basis of the entire course. In lecture 4 the one-atom ideal gas is treated, and in lecture 5 this is generalized to N atoms. In between is the first "aside" of the book, and this treats the case $N = 2$. It helps the student transition from the first lecture to the second and to become more comfortable with the multidimensional integrals that appear throughout the book. Such "repetition" is characteristic of the book and no doubt desirable for most students. In lecture 6 the ideal gas law $PV = Nk_BT$ is derived (the notation k_B is used for the Boltzmann constant). Three separate derivations are given, including a novel one that uses an arbitrary short-range gas-wall potential. Next the author carefully derives the equipartition theorem and its generalizations. The first part concludes with a discussion of how entropy is related to the partition function, thereby relating thermodynamics to statistical mechanics; the grand canonical ensemble for open systems is also discussed.

Part II is called "Separable Quantum Systems." It begins with lecture 10, which motivates quantum effects by considering the specific heats c_V of diatomic gases. For these, classical statistical physics predict that $c_V = 3k_B$, while experiments show that $c_V \leq 5k_B/2$. The discrepancy is resolved by using the discrete quantum mechanical energy levels for the harmonic oscillator and rigid rotor to recompute the partition function. This is then evaluated in the limits of high and low temperature. It would have been useful to conclude this section with lower and upper bounds on c_V, as predicted by quantum statistical mechanics. Next, specific heats of solids are calculated using the classical, Einstein, and Debye models. After an aside that summarizes quantum mechanics using the Dirac perspective, there is a very careful and detailed discussion of why one can replace averages over all quantum states of a system with an average over any set of basis states. Fermi–Dirac and Bose–Einstein quantum statistics are then introduced and there is an interesting historical discussion as to what exactly Planck did in his study of black-body radiation. The next aside treats the Kirkwood–Wigner theorem and part II concludes with chemical equilibria; the equilibrium constant is computed in terms of the partition function for reacting ideal gases.

Parts I and II treat very classical material, while the remainder of the book is more modern in flavor, beginning with part III: "Interacting Particles and Cluster Expansions." Lecture 15 has a general discussion of interacting particles and those potentials typically used to describe two-body interactions. Then, cluster expansions are introduced and their validity is discussed in detail. The virial expansion for a nonideal gas is derived and graphical notation for the virial coefficients is introduced. General methods for computing cluster integrals are discussed and specific examples are worked out. Distribution functions are treated next and part III concludes with the Debye–Huckel theory of the statistical mechanics of mobile charges.

Part IV is titled "Correlation Function and Dynamics." There is first a general discussion of correlation functions, including static and dynamic ones. It is shown how system symmetries can cause a correlation function to vanish. The next lecture establishes the time-invariance of averages for systems that evolve by Hamilton's laws of motion. After an aside that is devoted to a detailed but mathematically somewhat awkward derivation of the central limit theorem, the author treats the Langevin equation and Langevin's model of Brownian motion. This is related to statistical mechanics via the fluctuation-dissipation theorem; the Gaussian distribution of Brownian displacements is established. Limitations of the Langevin equation are discussed and a modified equation with a memory kernel is introduced. There is a brief qualitative discussion of interacting Brownian particles. The last two lectures in part IV treat the Mori–Zwanzig formalism and linear response theory. Part V (about 10% of the text) is more specialized and relates to the author's own research interest. It involves interacting Brownian motions that arise in light scattering experiments. The last part effectively brings together many of the preceding topics in the book.

Throughout the book, the author provides more than adequate mathematical detail. Mathematical simplifications are explained in words as well as equations. The author's style is to follow a mathematical derivation with a detailed discussion of the assumptions made and their range of validity. Usually physical counterexamples are given to illustrate situations where the model fails to be applicable. Sometimes such discussion is as long as the original derivation, but at the end of each lecture the author summarizes the main points in a brief "discussion" or "summary" section.

The mathematics are for the most part solid. There are a few places where improvements are possible. For example on page 118 there is the statement that $ln(N!) = NlnN - N$. Here it would be desirable to replace "=" by "$\sim$". In a couple of places we have $\int_{4\pi}(...)sin(\theta)d\theta d\phi$, and it is not clear whether 4π is the lower limit on the integral or its value. The symbol $\sim$ seems to be used to mean "of the order of magnitude" rather than "is asymptotic to." There remain quite a few typos: above (5.8) we see the identity $\exp(a+b)=\exp(a)+\exp(b)$.

The level of mathematics and physics seems appropriate for a graduate physics student but may be too difficult for a typical chemistry student. Overall this is a carefully written text that is worthy of consideration by instructors wishing to teach a basic statistical physics course, with some emphasis on cluster expansions, correlation functions, and dynamics.

CHARLES KNESSL
University of Illinois at Chicago

Principles of Applied Mathematics: Transformation and Approximation. Second Edition. *By James P. Keener.* Perseus Books, Cambridge, MA, 2000. xix+603 pp., hardcover. ISBN 0–7382-0129-4.

In *Principles of Applied Mathematics*, Keener sets an ambitious goal of providing a comprehensive and fundamental work describing the key methods of applied mathematics. As he states in the preface, "through the years it has become apparent that there are a group of tools that are essential to the analysis of problems in many disciplines. This book is about those tools." Indeed, the book is exactly as advertised. It covers a diverse set of topics beginning with finite-dimensional vector spaces and function spaces and finishing with singular perturbation theory. In between, Keener covers linear operator theory, the calculus of variations, complex analysis, transform and spectral theory, PDEs, inverse scattering, and asymptotic and perturbation methods—in essence, the core of classical applied mathematics techniques.

The book is well written and highly readable and covers a vast amount of material within its 600 pages. It includes a nice variety of problems for each chapter that are both instructive and geared to the beginning graduate student. A great advantage of this second edition is that many mistakes have been corrected in the problem sets and elsewhere. Additionally, the second edition includes new material that has been added to reflect more contemporary issues such as wavelet analysis.

Due in part to its attempt to be comprehensive and to include such a large amount of information, the coverage of many topics is terse and lacking in depth. In fact, many topics covered seem to be more refresher-like in nature than an actual pedagogical tool for instruction. Unless a beginning graduate student has an excellent applied mathematics background, he or she will most likely need supplementary materials for learning the covered topics. In this sense, the book is much better as a reference than as a text. Alternatively, it can be an excellent source of information for more advanced graduate students who already have a strong background in a wide variety of topics in applied mathematics.

My chief criticism of the book is in its lack of coverage of contemporary numerical tools for solving problems. I do not expect such a book to cover numerical methods. However, I think it would be a great help to provide MATLAB, Mathematica, or Maple routines that, for instance, solve some nonlinear differential equations like the Van der Pol oscillator. These codes could be very simple and could be easily incorporated into such a text. Further, the dynamics could be illustrated in the text as opposed to being simply described with an equation. Although an effort has been made to include problems that are to be done with the aid of the computer, it seems a bit artificial and forced. I think it would be more natural to include small bits of numerical tools throughout the book in the main text or in an appendix. This would greatly aid in tying together the principles of applied mathematics, which is the aim of the text. For today's applied mathematician, computational proficiency is absolutely crucial.

Despite my criticisms, I think it would be very difficult to find another book that covers so many important topics at such an advanced level. It goes well beyond the more simple undergraduate texts such as Kreyszig's *Advanced Engineering Mathematics*, Butkov's *Mathematical Physics*, or Arfken's *Mathematical Physics* and provides the higher level of mathematical sophistication needed for research-level projects. In this sense, Keener's book is a very valuable and excellent resource for those in applied mathematics and the engineering and physical sciences.

J. NATHAN KUTZ
University of Washington

Perturbation Analysis of Optimization Problems. *By J. Frédéric Bonnans and Alexander Shapiro.* Springer-Verlag, New York, 2000. $79.95. xviii+601 pp., hardcover. ISBN 0-387-98705-3.

Nonsmooth analysis has reached a certain level of maturity in which the variational properties of possibly nonsmooth, extended real-valued functions are understood as part of a coherent framework. Moreover, this is a field that has always had a close connection to issues in modern continuous optimization, and the interplay between nonsmooth analysis and optimization has benefited both fields. Thanks to the new theory developed, more and different kinds of optimization problems can now be fruitfully analyzed and, at the same time, developments in theory have been helpfully driven by needs discovered in practice. This mutually beneficial relationship is not automatic, however, and it needs nurturing from both sides. The book under review is a well-organized and timely offering that promises to effectively communicate some of the important new theory to readers with interests in continuous optimization.

The book begins with a succinct treatment of the fundamental results in nonsmooth analysis that support the rest of the material. The succinctness of this chapter is typical of these authors' treatment of material throughout and is one of the book's particularly attractive aspects. It does not attempt to be an exhaustive catalogue of results in this area, but instead it successfully presents one coherent path through a variety of interesting results. Everything is included that is needed to follow this path, and consequently to appreciate how nonsmooth analysis can develop in the context of optimization problems.

The background material is followed by a chapter developing first- and second-order optimality conditions for the general constrained optimization model to be used throughout the book:

$$\min_{x \in Q} f(x) \qquad \text{subject to } G(x) \in K.$$

It is assumed that f and G are smooth and that the set K is closed and convex. This model covers, in particular, nonlinear programs, semidefinite programs, semi-infinite programs, composite optimization models, and, because it is developed in a Banach space setting, optimal control and variational problems. All the nonsmoothness in this model is concentrated in the set K, which is a useful tactic for illustrating the effects of nonsmoothness on the analysis. The development of the optimality conditions is nicely organized in terms of a quadratic growth condition, which, when the model is sufficiently regular, can be characterized by second-order conditions on the data.

The heart of the book is the sensitivity analysis in Chapter 4, carried out on a parameterized version of the general optimization model. The authors are themselves major contributors in this area, and the results in this chapter reflect their approach. This involves the "directional analysis" of scalar perturbations along a fixed direction, and the results here are organized around this concept. The focus is on computing upper and lower estimates of the optimal value function, using directional regularity assumptions together with second-order sufficient conditions. Different order expansions of the optimal value function, nearly optimal solution selections, and optimal solution selections are obtained under different sufficient conditions. There are other approaches to sensitivity analysis and other kinds of results, but the authors wisely focus on a single coherent point of view. The result is a unified treatment and a reasonably complete sensitivity analysis which should be appreciated by many readers.

The book ends with two chapters devoted to special cases of interest that fit the general model. The first of these chapters includes sections on variational inequalities, nonlinear programs, semidefinite programs, and semi-infinite programs, and the second chapter is devoted to optimal control problems. These chapters are largely self-contained, so readers with particular interests in any of these areas could first open the book here.

This book should appeal to readers with interests across the full spectrum from nonsmooth analysis to optimization, because it provides a clear illustration of some of the connections between these fields. The authors have done their part to nurture this important relationship, and it is up to the readers to follow their lead.

ADAM B. LEVY
Bowdoin College

Phase Resetting in Medicine and Biology. *By Peter A. Tass.* Springer-Verlag, New York, 1999. $99.00. xiii+329 pp., hardcover. ISBN 3-540-65697-9.

Are medical treatments just an electrical pulse away? Implantable electrical stimulators are increasingly being used to manage troublesome medical problems ranging from the prevention of life-threatening cardiac arrhythmias and epileptic seizures to the reduction of the morbidity associated with chronic pain and movement disorders. Physicians currently adjust the properties of the electrical stimuli (e.g., amplitude, pulse width, frequency) using a trial and error approach. However, for any oscillator it is possible to measure a phase resetting curve. This curve summarizes the response of an oscillator to brief perturbations of a given magnitude as a function of the phase of the oscillator at which they were administered. Obviously, theoretical insights into phase resetting would be very useful to help physicians rapidly optimize the beneficial effects of electrical stimuli.

This book presents a theoretical study of the spatiotemporal dynamics of populations of identical phase oscillators subjected to noise and stimulation. The author demonstrates that it is possible to obtain meaningful insights into the behavior of these networks using techniques that are quite familiar: the Fokker–Planck equation and Fourier analysis. Topics discussed range from the phase resetting properties of populations of uncoupled phase oscillators (Chapter 2) to the spontaneous formation of regions of synchronized activity (clusters) within large populations of coupled phase oscillators (Chapters 3–5) to the effects of single pulse perturbations (Chapter 6) and periodic stimulation (Chapter 7). The discussion is clearly written and the inclusion of many figures makes the development of the ideas easy to follow. Each chapter has a final summary attached to it. These summaries are so well written that a nonmathematically oriented reader can readily understand the message by simply reading the summaries.

The final chapters of the book (Chapters 8–10) discuss applications of the theory for the analysis of EEG/MEG data and for improving brain stimulation protocols. I am not enthusiastic about these chapters since the discussion is very hypothetical and the applications to real data rather weak. In cardiology the success of phase resetting analysis to account for certain cardiac arrhythmia stems from the strong interplay between prediction and observation. It is disappointing that the author was unable to make a similar case for the use of phase resetting analysis in brain stimulation. Indeed, the experimentally oriented reader will be left wondering whether all of the beautiful theory was really worth the effort.

All said and done, it must be admitted that phase resetting theory has had little, if any, impact on clinical medicine. The current use of electrical stimulators in medicine does not take advantage of the phase resetting properties of electrical stimuli. This text does little to change this situation. For example, the book cover promises improvements for the brain stimulation techniques for neural diseases. The message turns out to be a single short paragraph (p. 274), which I strongly doubt that neurologists and neurosurgeons will find of much value. Thus we have the field of phase resetting: much promise, no deliverables at present, but the future remains bright.

This book will disappoint biologists and physicians looking to obtain an introduction to the application of phase resetting to biology and medicine. On the other hand, this text, in particular Chapters 2–7, is clearly a must read for those interested in the spatiotemporal dynamics of populations of oscillators.

JOHN MILTON
The University of Chicago

Grid Generation Methods. *By Vladimir D. Liseikin.* Springer-Verlag, Berlin, 1999. $87.00. xvi+362 pp., hardcover. ISBN 3-540-65686-3.

Grid generation is an important and difficult problem in applied mathematics. I approach reviewing this book not as an expert in grid generation methods but as a user of grid generation techniques. A number of good books on grid generation have been written in the last several years. In reviewing this book I will compare it to the recent effort by Knupp and Steinberg [4]. The two books follow similar outlines and cover many of the same topics. The book by Liseikin is significantly more rigorous and presents the material in a more general setting. At times this leads to a loss of clarity. One asset of the book is the extensive bibliography (with historical footnotes and comments) that ends each chapter.

Liseikin views grid generation as a critical part of discretizing partial differential equations. The primary focus of the book is on structured grid methods, but he includes material pertaining to unstructured grids in each chapter and devotes the last chapter to some unstructured grid algorithms.

The first chapter is a general introduction to grid generation. The next two chapters provide the notational and geometrical framework for his presentation. In the next six chapters he presents a series of approaches to grid generation. The tenth chapter, "Comprehensive Method," is the author's method. The final chapter is devoted to some unstructured methods.

The first chapter begins with arguments on the need for efficient and robust grid generation methods, a brief history of grid generation efforts (more historical information can be found throughout the book), and the problems that currently confront the field. He then proceeds to discuss and define the basic concepts related to grid generation and provides an outline of the book.

In the second chapter the author introduces much of the notation that will be used throughout. The notation is presented in the context of transformations between the "logical region" where the computations are carried out and the "physical domain" where the partial differential equations are specified. As in all the chapters, Liseikin seeks to provide a geometric understanding of the algebraic formulas, although I would have preferred more figures. In the final two sections he describes the impact of the grid transformations on various partial differential equations. Here, he improves on the effort by Knupp and Steinberg [4].

Measures of grid quality are developed by the author in the third chapter. Concepts from differential geometry essential to grid generation are introduced, beginning with the geometry of curves and surfaces. The author stresses the importance of using geometric invariants in his definitions of grid quality measures. He divides his grid quality measures into three groups: measures on grid lines, measures on grid faces (in three dimensions), and measures on grid cells. In each case the list is extensive and many of the formulas for the grid measures, in terms of the geometric invariants, were developed by Liseikin.

The next six chapters describe the grid generation techniques currently in use. Chapter 4 focuses on stretching methods. Stretching is probably the simplest and most natural grid generation method. As the name implies, the uniform "logical grid" is stretched or contracted in each direction via univariate functions. The author couples his presentation of stretching with singular perturbation problems. In these problems appropriate stretching functions can significantly improve the solution in boundary and interior layers. He presents such stretching functions for a wide variety of situations. I found this to be a nice overlap between analytical and numerical methods.

In the fifth chapter algebraic grid generation via transfinite interpolation is presented. The boundary of the logical region is mapped to the boundary of the physical domain using blending functions. As a grid generation user, I found this chapter to be the most difficult to understand. I first needed to review the comparable chapter in Knupp and Steinberg [4] before I understood what the author was presenting. For experts in the field the additional abstraction provided by the author may be an asset.

The sixth chapter focuses on grid generation via the solution of partial differential equations. Beginning with Laplace's equation in two dimensions, the author analyzes the properties of grids produced using this approach. More control over grid characteristics is obtained by solving Poisson systems. As a specific example he looks at generating grids with near orthogonal grid lines (in two dimensions) near the domain boundary. He then looks at the increased flexibility in controlling grid behavior that can be obtained by solving the biharmonic equation. In the final section grid generation methods that arise from solving hyperbolic and parabolic equations are discussed. These methods are generally more efficient but at the possible cost of grid quality.

Chapter 7 concerns adaptive grid generation for both steady and unsteady problems. Most of the chapter focuses on the use of the equidistribution principle beginning in one dimension and proceeding to higher dimensions. The equidistribution principle selects a grid by attempting to distribute some quantity (called a control function), such as the error, equally over the elements of the grid. Various control functions are presented and the method is applied to unsteady problems. Significant, recent mesh moving strategies were overlooked, such as the work of Cao, Huang, and Russell [2]. Other adaptive approaches used extensively in the adaptive refinement community [1] were not discussed at all.

Grid generation based on variational principles is presented in Chapter 8. Here the author introduces the idea of grid optimality into the grid generation process. He begins with a review of the calculus of variations and the Euler–Lagrange equations. He then applies these equations to functionals derived in Chapter 3 as grid quality measures and to functionals involving the solution residual (as a measure of the error). The author also rederives the method involving Laplace's equation of Chapter 6 by using an appropriate functional involving grid smoothness. This leads back to the equidistribution principle of Chapter 7. The final three sections consider other possible functionals that lead to grids with certain desirable properties (such as grids that align with solution features) and combinations thereof.

In Chapter 9 Liseikin discusses grid generation methods for curves and surfaces, especially as they are considered as boundaries of regions that need grids. The initial discussion in [4] of the same concepts is clearer. While many of the methods for planar domains in Chapters 4–8 could be used in this case, adjustments must be made to include the properties of nonplanar curves and surfaces. Basic formulations of the problem for curves and surfaces are developed in sections 9.2 and 9.3, respectively. In the next two sections a method based on solving Beltramian systems is proposed and analyzed. These systems are generalizations of Laplace's equation of Chapter 6. Liseikin shows that additional control over the grid can be achieved by utilizing the Poisson equation method of Chapter 6 in this context.

The tenth chapter presents the "comprehensive method," which is the method due to the author. The chapter begins with a discussion of the "big grid code" due to Godunov and Prokopov [3] and Thompson, Thames, and Mastin [6] including a list of its shortcomings. The author's proposal is to use a "variational technique for the generalization of uniform grids on hypersurfaces with the help of a generalized smoothness functional whose minimization produces harmonic mappings." The remainder of the chapter provides details on how this is done.

The author reviews two unstructured grid generation strategies in the final chapter, "Delauney and Advancing Front." In section 11.2 he discusses issues specific to unstructured grids. The third section outlines Delauney triangulation in two dimensions and briefly presents a generalization to three dimensions. His discussion of the advancing front method is even shorter. He leaves other unstructured methods such as the work of Shephard et al. on octrees [5] to the bibliography (which is extensive in this chapter). This chapter provides only a taste of unstructured grid generation.

This book by Liseikin contains an extensive presentation of structured grid generation methods. Its presentation of unstructured grid methods is much briefer. It supplements, updates, and adds to the earlier work of Knupp and Steinberg [4]. As such it would be a valuable asset to both experts in the field and users of grid generation algorithms who want a better understanding of the tools they use.

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Peter K. Moore
Southern Methodist University

Autoparametric Resonance in Mechanical Systems. *By Ales Tondl, Thijs Ruijgrok, Ferdinand Verhulst, and Radoslav Nabergoj.* Cambridge University Press, Cambridge, UK, 2000. $59.95. x+196 pp., hardcover. ISBN 0-521-65079-8.

An autoparametric system, as defined in this book, is a mechanical system consisting of two coupled subsystems, one of which (the primary system) is able to oscillate while the other (the secondary system) remains at rest. Such a solution is known as semitrivial, and much of this book is devoted to the questions of stability of the semitrivial solution and bifurcations that take place from this solution when it loses stability. It appears that it is frequently desirable, from an engineering standpoint, for the semitrivial solution to be unstable in such a way that as the secondary system begins to oscillate, it draws energy from the primary system and prevents its oscillations from becoming unbounded. In fact, secondary systems of this type can be deliberately added to a mechanical system to serve as "vibration absorbers." Typical examples considered in the book are spring- mass systems with attached pendula, rotors with elastic supports, and heave-pitch-roll models for ships.

Instead of viewing an autoparametric system as mechanical, it could be defined as a system of differential equations capable of modeling such a mechanical system (and equally capable of modeling nonmechanical systems, such as electrical networks, exhibiting the same behavior). Although no nonmechanical examples are presented in this book, the majority of the text is devoted to the analysis of the differential equations, and relatively little to the modeling process leading to those equations.

The manner in which the material is presented seems to pull in two directions at once. On the one hand, there are many specific systems for which detailed results are presented, in a style suited for engineers. (For instance, stability conditions are given as complicated expressions involving many parameters having specific engineering relevance.) In these examples, the mathematical details and proofs are almost entirely lacking and are replaced by sketchy treatments with references to the literature or to Chapter 9, a grab bag of mathematical techniques from perturbation theory and dynamical systems. On the other hand, there are a few general models of a more mathematical nature, defined by such things as the eigenvalues of the linear part and the functional form of the nonlinearities without regard to specific engineering details. These are discussed in a manner that could only be understood by a mathematician having at least some familiarity with dynamical systems; notions such as genericity, invariant tori, Silnikov bifurcations, and Melnikov integrals are mentioned in passing, but with few details. Chapter 9 itself, which promises to be the most interesting to a mathematician, is actually only a sketchy introduction to the topics covered in a first book on dynamical systems.

Probably the most valuable use of this book will be as a guide to the literature. An engineer may treat the book as a (presumably nearly complete) annotated list of the existing papers on autoparametric systems, together with a sketchy introduction to the relevant mathematical literature. A mathematician in dynamical systems will find a collection of potentially interesting applications, many of them containing open problems. A graduate student may regard the book as a long set of exercises with extensive hints (and possibly a source of thesis topics). No one will find a satisfyingly complete treatment of any one part of the subject, but that is probably not a defect given that this is a short book on a narrow and (at least potentially) rapidly changing topic.

JAMES MURDOCK
Iowa State University

Applied Functional Analysis. Second Edition. *By Jean-Pierre Aubin.* Wiley, New York, 2000. $99.95. xvi+495 pp., hardcover. ISBN 0-471-17976-0.

The author begins the preface of this book with a statement, "Yet, another book on functional analysis! Yebfa!, would exclaim a computer scientist in his or her exotic language." Thus the author anticipates the very first question that comes to one's mind when noting that this book has appeared some 20 years after his first edition on the subject. The author's justification is very understandable to anyone who has written textbooks on this subject (including the reviewer, who has written two). "The first edition of this book reflected my personal experience at the time derived from numerical analysis and partial differential equations, and later, from mathematical economics. After two decades my views have evolved and my experience has broadened, my teaching of functional analysis to students of Université Paris-Dauphine evolving year after year. I could not resist both the pleasure and the pain of divulging to the young students what was continuously going on at their level, on the research front." This new edition covers the functional analytic background necessary to approach quickly a number of new subjects not often found in a course on functional analysis. These include a thorough introduction to Sobolev spaces and approximation procedures in spaces of functions, Fourier transforms in Sobolev spaces, set-valued analysis and convex analysis, spectral theory of compact operators, and then a detailed analysis of the theory of boundary value problems. There is a chapter on differential-operational equations and semigroups of operators and a chapter on viability kernels and capture basins.

This book, nearly 500 pages in length, is both a textbook and an excellent reference. A set of well-conceived exercises for each chapter is given as an appendix. Also collected at the end of the text is a large section titled "Selection of Results," where the principal theorems and results are collected for the convenience of the reader. Here one finds what the author views as the principal theorems and results covered in each chapter restated and summarized in the order in which they are covered in the text, it being understood that less important properties or more general statements are covered in the main body of the work.

To the student reader of this book, it must first be explained that this is not exactly an introduction to the subject. The book requires that the reader have a mastery of the fundamental notions of topology in metric spaces and vector spaces. Given this, the book is self-contained.

There are 16 chapters in the book, divided into three basic parts. The first part, consisting of the first five chapters, is a well-done treatment of what might be called linear functional analysis. It covers the classical theory of projections and Hilbert spaces and some topics fundamental to optimization theory. The second part, Chapters 6–9, is on examples of Hilbert spaces, specifically Sobolev spaces and distributions, and deals with some methods of approximation. The remainder of the book deals with an introduction to set-valued analysis with applications to convex analysis, optimization, the spectral theory of compact operators, Hilbert–Schmidt operators, and a study of boundary value problems and variational inequalities. The final chapter covers first-order partial differential equations, with accounts of Hamilton–Jacobi equations, first-order systems, and distributed boundary data.

I find the book well written and presented in the organized, compact style we have learned to expect from the author. The book contains a number of detailed treatments of subjects not normally found in texts on functional analysis put together in a logical fashion that makes the main results readily accessible. I believe the book will provide an excellent reference to students and applied mathematicians working in a broad number of areas. It is certainly a welcome addition to the literature. It will not fit the bill of all existing courses on applied functional analysis, because the choice of topics (as is true of all textbooks) reveals the personal biases and experiences of the author. But this book will be a valuable reference for most of them.

J. TINSLEY ODEN
The University of Texas

Lie Groups. *By J. J. Duistermaat and J. A. C. Kolk.* Springer-Verlag, New York, 2000. $48.00. viii+344 pp., softcover. ISBN 3-540-15293-8.

The book under review constitutes the first of a projected two-volume work devoted to the basic theory of Lie groups and Lie algebras. The present volume starts out by introducing the key concepts in Lie group and Lie algebra theory. The remaining three chapters then concentrate on compact Lie groups and proper actions. Other aspects of the theory will appear in the subsequent volume; indeed, there are many forward references already included in the present text.

Even though it claims to require only basic geometric and analytic prerequisites, this book is not an elementary text. The mathematical sophistication and demands of rigor make this appropriate only for the truly dedicated student who wishes to acquire a deep and rigorous foundation in Lie group theory. Despite the level of exposition, I found the book well written and eminently readable. While the initial sections include some very nice, elementary examples to illustrate the theory—and include additional less commonly known details in many cases—the later, more complicated material would be more easily digested if a comparable range of simple illustrative examples were included. Each chapter concludes with several pages of excellent historical remarks and references to the original literature. The authors have included a wide selection of exercises, many of which also require significant mathematical sophistication and effort.

Highlights include an innovative proof of the global form of Lie's third fundamental theorem (every Lie algebra corresponds to a unique connected, simply connected Lie group) that avoids the detailed Lie algebraic structure theory, but instead relies on the geometric theory of infinite-dimensional Banach Lie groups and the characterization of the Lie group as a suitable equivalence class in the space of paths in the Lie algebra. The second chapter contains a wealth of details on the orbit stratification of proper actions not previously available in texts. While this volume does not get up to the Killing–Cartan classification of semisimple Lie groups, its treatment of the structure theory of compact Lie groups and their representations introduces many of the important tools and culminates in the Weyl integration and covering theory. The final chapter develops the highest weight classification of representations of compact Lie groups, culminating in the Peter–Weyl theorem that generalizes Fourier analysis to arbitrary compact groups and the Borel–Weil theory that realizes each representation on the space of sections of a certain line bundle.

The book is very much in the pure mathematical mold, concentrating on the inner beauty and symmetry of the subject. As an advanced mathematical monograph, it forms a welcome addition to the literature on Lie group theory. However, it is of less immediate relevance to applied practitioners of Lie theory. Thus, the very comprehensive treatment of the theoretical foundations and ramifications of the justly famous Peter–Weyl theorem in the representation theory of compact groups never exposes their tremendous impact in quantum mechanics or in the theory of special functions. Similarly, while the basic theory on transformation groups is reviewed, the text then goes off into rather sophisticated results on compact actions, slices, orbit stratifications, and so on. One would never know that Lie groups can be used to solve differential equations arising in a broad range of applications or to classify differential invariants such as curvature, which is of importance in many current applications in geometry, physics, mechanics, and image processing.

For the right audience, the book could be profitably adopted in an advanced topics course on pure Lie group theory. The prerequisites would be a sufficiently mature and motivated group of students and a sufficiently dedicated instructor. However, as an applied mathematician, my own inclination would be to devote such a topics course to genuine applications of Lie theory, and this book would not be as appropriate. In summary, the book would not be the text to recommend to someone interested in applications of Lie groups in physical systems and applied mathematics, but it does form a solid and praiseworthy account of the more advanced and fascinating realms of mathematical Lie group theory.

PETER J. OLVER
University of Minnesota

Numerical Bifurcation Analysis for Reaction-Diffusion Equations. *By Zhen Mei.* Springer-Verlag, New York, 2000. $84.00. xiv+414 pp., hardcover. ISBN 3-540-67296-6.

Reaction-diffusion equations $\frac{\partial u}{\partial t} = D\Delta u + f(u,\lambda)$ arise as mathematical models in many scientific problem areas, such as in chemical reactions, biological systems, population dynamics, or nuclear reactor physics. At least since a pioneering article by A. Turing in 1952, it has been well known that the solutions of these systems often exhibit spontaneous formation of various spatial or spatial-temporal patterns. These changes under variation of the control parameters reflect a wealth of possible bifurcation phenomena, and their study has now become the topic of a large and ever growing literature. A wide spectrum of mathematical approaches has been applied in these studies, but for practical applications computational methods frequently remain the only feasible avenue. This monograph aims to present an overview of the numerical analysis of bifurcation problems in reaction-diffusion equations. It appears to have grown out of the author's work on his habilitation thesis, that is, the second thesis required in Germany for entrance into an academic teaching career.

The book consists essentially of two almost equal parts although they are not identified as such. The first part covers Chapters 2–8 and provides background material from bifurcation theory and related computational aspects required in the second part. This begins with a summary of numerical methods for continuing solution branches of stationary, finite-dimensional equations $G(x,\lambda) = 0$, for the construction of test functions to detect certain bifurcations on such solution branches, and for switching branches at simple bifurcation points. Then some basic properties of symmetries are presented followed by an introduction to Liapunov–Schmidt reductions and to the principal results of center manifold theory. Finally, numerical aspects relating to (quasi-) periodic solutions near a homoclinic orbit of equations $\dot{x} = f(x,\lambda)$ are addressed.

As is not surprising with such a wide range of material, the presentation in these seven chapters is often fairly brief and refers the reader to the literature for further details and motivations. While most of this material is available in a number of texts and monographs, a distinguishing feature may be an emphasis on computational aspects and some use of informal algorithms for defining various methods.

While in the first part reaction-diffusion equations are mentioned only occasionally, the remaining chapters, 9–16, concentrate on the topic in the title of the book and form its principal part. The presentation is mainly centered on systems of one or two equations in one or two space dimensions and, in the latter case, usually with the unit square as the domain. Polynomial growth conditions are used for the reaction term $f(u,\lambda)$ to ensure classical solutions. In the case of scalar equations, interest centers on the preservation of multiplicities in the discretized problems and the use of continuation for tracing solution branches. For systems of two equations on a square, Liapunov–Schmidt reduction is used to analyze the diagrams at simple and double bifurcation points. Then, for the same class of systems, normal forms are developed that cover both reducible and irreducible representations of the symmetry group. The next two chapters concern steady/steady state and Hopf/steady state mode interactions for reaction-diffusion equations. From here on the presentation appears to follow mainly the approaches of the group associated with K. Böhmer at the University of Marburg, Germany, to which the author belongs. This emphasis on the work of that group is particulary evident in the last three chapters about the influence of boundary conditions on bifurcation for reaction-diffusion equations. After an analysis of the properties and spectrum of the Laplacian under variable boundary conditions, variation of the steady state bifurcations along a homotopy from Neumann to Dirichlet boundary conditions for scalar reaction-diffusion equations are discussed. Then, by treating a homotopy parameter in the boundary conditions as second parameter, the generic bifurcation behavior and stability of the solution branches are analyzed.

The presentation of the material is very readable but marred in places by an awkward style and grammatical lapses. This monograph certainly brings together a wide range of material on the numerical analysis of bifurcation phenomena of reaction-diffusion equations and should be attractive to anyone interested in this area.

WERNER C. RHEINBOLDT
University of Pittsburgh

Binary Digital Image Processing. *By Stephane Marchand-Maillet and Yazid M. Sharaiha.* Academic Press, New York, 1999. $74.95. xxx+251 pp., hardcover. ISBN 0-12-470505-7.

Many important classes of scenes contain objects that strongly contrast with their background, for example, dark objects on a light background. In an image of such a scene, object points have high values (where the value represents darkness) and background points have low values. Thus the content of such an image can be effectively represented by a two-valued function, having (say) value 1 at object points and value 0 at background points. One of the most common classes of high-contrast "scenes" are document pages; here the "objects" are usually characters (in text) or lines (in drawings).

For computer processing, images are represented in digital form as arrays of numbers; the elements of a digital image are called pixels, and their values are called gray levels. If the image represents a high-contrast scene, the values of its pixels can be taken to be 0 and 1; such a digital image is called BINARY. Connected sets of 1's in a binary digital image represent objects in the scene. In general, these objects can be of any size or shape, but in many situations they consist of elongated ("ribbon-like") parts, e.g., if the objects are characters or lines in a document image.

This book deals with the representation and processing of binary digital images and with the definition and computation of geometrical properties of objects in such images. The concepts and methods treated in the book's eight chapters can be briefly summarized as follows:

(1) Basic topological and metric concepts related to digital images: triangular, hexagonal, and square arrays (lattices); neighborhoods; digital arcs and closed curves; connected components and borders; discrete distances.

(2) Basic concepts of discrete geometry: chain codes and straightness; convexity; curvature; circularity; parallelism; and orthogonality.

(3) Graph-theoretic algorithms for shortest-path and minimum spanning tree problems and their application to graphs defined by pixel adjacencies in digital images.

(4) Methods of image digitization; data structures for storage of binary digital images; binary data compression.

(5) Computation of discrete and Euclidean distances in digital images.

(6) Connected component labeling; "noise cleaning" and contour smoothing; decomposition of connected components into parts; computation of geometric properties of connected components.

(7) Thinning of elongated objects; processing and approximation of "line images" composed of digital arcs and curves.

(8) Examples, including line-drawing, fingerprint, and handwriting images.

There is also a bibliography of 179 references, including many basic papers and books on digital images, digital geometry, and mathematical morphology.

It should be mentioned that two earlier books exist on binary (digital) image processing. The first [1], which is nearly 20 years old, is a collection of reprints covering binary image models and quality metrics; image thresholding and reproduction (including halftoning); binary image coding, enhancement, and scaling; and applications. The second book [2], after introducing gray-level image processing, has chapters on gray-tone to two-tone conversion; two-tone image preprocessing (including brief discussions of basic geometrical concepts, as well as approximation, scaling and rotation, smoothing, and enhancement); two-tone image coding and compression; shape analysis, representation, and description; and applications. Thus the earlier books have relatively little overlap with the present book.

As we might expect from its authors' credentials, the strength of this book lies in its clear and systematic treatment of discrete geometric concepts and algorithms. This material should be useful to students, practitioners, and specialists and should be of interest to readers of the present journal because of its mathematical nature.

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Azriel Rosenfeld
University of Maryland

Measurement Errors and Uncertainties: Theory and Practice. Second Edition. *By Semyon G. Rabinovich*. Springer-Verlag, New York, 2000. $44.95. xii+296 pp., hardcover. ISBN 0-387-98835-1.

This is a compendium of descriptive properties of measurement errors arising in the physical sciences. The main types of measuring devices discussed in this book are of classical type: scales, thermometers, transducers, voltmeters, etc. Digital instruments are mentioned in Chapter 2, but the general statistical issues related to errors arising in highly accurate measurements of digital type (like network traffic data) are not discussed. An exception is the brief example of the calculation of errors of digital thermometers in Chapter 9.

The author, who worked for a number of years in the Soviet All-Union State Research Institute of Metrology, presents his unique view of estimation of measurement errors and uncertainties. The reader of this book must be aware that the definitions (given mainly in Chapters 1–3) are mainly idiosyncratic and may not coincide with the ones commonly used in the West. The best source for these definitions is provided by [1], [2], and [3]. Also a survey article, [4], is very relevant for contrasting different statistical methodologies as applied to measurement errors.

Here are some differences in terminology: The measurement error is agreeably defined on page 2 as "the deviation of the result of measurement from the true value of the measurable quantity." A bit of confusion is added by the fact that this error can be "expressed in absolute or relative form." More confusion is caused by the definition of the absolute error as the difference between the result of measurement and the true value of the measurable quantity (and not of the absolute value of this difference). This notion of uncertainty is not the one that is commonly accepted in this country. According to the author it is "an interval within which a true value of a measurand lies with a given probability." Thus, uncertainty becomes the function of the coverage probability, and this is in contrast with the American concept. Indeed the definition given in [2] is as follows: "The uncertainty of the result of measurement generally consists of several components which. . .may be grouped into two categories according to the method used to estimate their numerical values: A. those which are evaluated by statistical methods, B. those which are evaluated by other means." Uncertainty evaluated by statistical methods is essentially the standard deviation of the error distribution. The author (p. 68) criticizes this definition as describing the errors not through their properties but rather by the method employed to estimate them. However, the suggested terminology, *absolutely constant elementary errors* and *conditionally constant elementary errors*, has hardly a chance to gain popularity.

Dr. Rabinovich argues strongly against the possibility of measuring random quantities (p. 15). According to him, “these quantities, as such, do not have a true value, and for this reason they cannot be measured.” Needless to say, this point of view is in conflict with the modern statistical methodology, with now computationally strong Bayesian techniques, which are based on the concept of random parameters. It seems that the author contradicts himself when he discusses in Chapter 5 the estimation of elementary (random) errors. Incidentally, the uncertainty assigned by Bayesian methodology is conveniently expressed by the posterior distribution, and the latter concept is not touched upon in the statistical part of the book.

This part, which also has a classical flavor, consists largely of Chapters 4–7. Notably the paramount propagation-of-errors method is critically reviewed in Chapter 6. The author finds shortcomings in this method, as the Taylor series approximation may be inadequate because of the small number of terms included. The traditional way of combining uncertainties [1] is criticized, as it may lead to an “overestimated estimate” (p. 147). This is somewhat ironic, as elsewhere the author argues in favor of conservative estimates for uncertainties. He suggests a new method which, however, assumes that the degrees of freedom for the systematic error component are known.

The examples in Chapters 5, 6, 7, and 9 form, from the reviewer’s point of view, the most interesting part of the book. Chapter 5 contains an illuminating contribution as an example of uncertainty calculation in voltage measurements by a pointer-type voltmeter. It is shown that the uncertainty can decrease when more accurate information about the properties of measuring devices is employed. Chapter 9 also gives specific examples of error calculations for several types of measuring instruments (electric balances, voltmeters, etc.). It is worth noticing that in most of these examples, the traditional methods and the author’s methods lead to almost the same numerical answers.

Chapter 8 deals with the important problem of combining information from the results of measurements in different laboratories, centers, etc. A question of fundamental importance in the analysis of such data, particularly when certifying standard reference materials, is how to form a best consensus estimator of common parameters and what uncertainty to attach to this estimate. It is worth mentioning that the National Institute of Standards and Technology (NIST) uses an estimation equation approach due to Mandel and Paule [5], [6] to obtain the weighted means statistics, which are also discussed in Chapter 8. Problems of this kind are becoming more important as the meta-analysis of data obtained from different studies is required by modern technology.

The monograph concludes in Chapter 10, with the calibration problem being the main subject.

This second edition is a revision of the first edition, which was a translation from the Russian by M. Alferieff. Unfortunately, some typos are left in this edition. For example, “permissable” deviations and errors occur on several occasions, “Outlying Results” is supposed to mean “Detection of Outliers,” and “the squares-root sum method” (p. 169) is merely the formula for the variance of the sum of independent random variables.

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Andrew L. Rukhin
National Institute of Standards and Technology and University of Maryland

Mathematical Analysis of Viscoelastic Flows. *By Michael Renardy.* SIAM, Philadelphia, 2000. $35.00. x+104 pp., softcover. ISBN 0-89871-457-5.

In recent years, excellent books have appeared dealing with the mathematical aspects of viscoelastic fluid flow, i.e., on topics such as equations of state, stability, existence, uniqueness, and bifurcation [1], [2], [3], [4]. However, for the beginner (or even a more advanced researcher) these references often provide more information than necessary to quickly see what is new, exciting, or just plain hard in the subject. Here is where Renardy's lecture notes fit in. They provide a rapid overview of many of the basic mathematical and physical issues associated with viscoelastic fluid flow in an exceptionally clear and accessible manner. Nomenclature and ideas of the subject are introduced in such a way that even a nonexpert can quickly gain understanding. Furthermore, mathematical issues and experimental results are interwoven in a fashion convincing the reader (I hope) that the mathematical models analyzed really do say something about real fluids. Renardy alternates between rather applied sections where the emphasis is on deriving a simple set of equations to describe a phenomenon and analytical sections where more abstract mathematical issues are addressed. However, whether the topic is derivation of a simple set of equations for jet breakup or explanation of the symbol of a differential operator, the clarity of exposition (and Renardy's wit) are never lost.

I am happy to recommend Renardy's notes as a snapshot of the state of research in viscoelastic flow in the year 2000.

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Marshall Slemrod
University of Wisconsin-Madison

Arbitrage Theory in Continuous Time. *By Tomas Björk.* Oxford University Press, Oxford, UK, 1998. $45.00. xii+312 pp., hardcover. ISBN 0-19-877518-0.

Mathematical finance has been a rapidly growing area of application of mathematics, and in particular of stochastic calculus, since Black and Scholes derived their famous option pricing formula. This formula is widely used in today's financial world, eventually winning them the Nobel Prize in economics. As is true for all young fields there has been a distinct lack of good introductory texts and textbooks on the topic. Tomas Björk's book *Arbitrage Theory in Continuous Time* is one of several recent publications intended to close that gap.

In mathematical finance we are concerned with the pricing of financial assets that are known as financial "derivatives" because their value depends on that of some "underlying" stock. One important example is the European call option which guarantees the holder the right to buy a certain amount of the underlying asset from the derivative issuer at a fixed price and at a predetermined exercise time T. Other widely traded derivatives include American or European call or put options, forward rate agreements, and bonds or bond options.

These derivatives generally allow holders to guard themselves against the financial risk associated with a future commitment related to the underlying stock. Part of the risk is passed on to the issuer of the derivative, and so it is not surprising that this kind of insurance usually comes at a price. This leads us straight to the two core questions of the book:

- What is a sensible price for the derivative in terms of the underlying asset and how does this price evolve through time until the exercise date?
- How can the issuer of the derivative "hedge" the claim, meaning how can she ensure that she will be able to meet the future claim against her?

In order to tackle these questions two basic steps have to be taken. First, we need a model for the evolution of the underlying asset and a hypothesis of how the price of the derivative might depend on it. Second, we need some assumptions about the market's behavior.

Given the enormous number of factors influencing any stock price our model of choice is of a stochastic nature. Björk starts out to build intuition with the discrete time binomial model, in which the stock price increases or decreases at each time step by a certain factor according to some probabilities. However, the focus of the book is almost exclusively on continuous time models, and so on the diffusion limits of the discrete model. Thus, after a brief, self-contained introduction to stochastic calculus we move on to describing the price S of a risky asset by the stochastic integral equation symbolically written in its differential form as

$$dS = S\alpha dt + S\sigma dW.$$

Here, $\alpha = \alpha(t, S)$ is called the local mean rate of return, and $\sigma = \sigma(t, S)$ the volatility. W stands for a standard Wiener process (Brownian motion). Thus, the last term is to be understood as a stochastic Itô integral. The volatility is zero for a risk-free and therefore deterministic asset, which is the case if we are investing money in a bank. We write $dB = rBdt$, where r is the interest or spot rate.

The main assumption made about the market's behavior is that "there is no free lunch," or in other words, no arbitrage. This condition, which already gets us surprisingly far, summarizes the fact that there is no risk-free way to make a profit above the general interest rates. If a deterministic "money making machine" existed, surely this book would have been written about that—and become an instant bestseller!

Many standard financial derivatives are, like the underlying stock itself, traded on the market in large quantities. In this case, it is not hard to see that the two questions we have asked can be answered at the same time if we assume further that the price of the derivative $\Pi(t) = F(t, S(t))$ is only a function of the current value of S, and likewise that our claim is of a simple nature, given by $\Phi(S(T))$ and depending only on the stock price S at the exercise date T.

By Itô's famous formula, the equivalent of the fundamental theorem of calculus in the stochastic setting, we may obtain the stochastic evolution equation for Π. We can then build up a portfolio of S and Π, which means a collection of certain quantities of the stock and the derivative (note that holdings may and will be negative!), such that we have for the value V of the portfolio $V(T) = \Phi(S(T))$ with probability 1. To rule out arbitrage possibilities it is clear that this stipulates $\Pi(t) = V(t)$ for all $t \leq T$.

Such a portfolio, which is "replicating" the claim because it is formally equivalent to it, may be constructed by choosing the quantities in the portfolio in such a way that the stochastic term for V vanishes. The equation is then deterministic and therefore has to have the same dynamics as B, again because of the no arbitrage assumption. These conditions taken together finally imply that the pricing function F solves the Black–Scholes PDE

$$\begin{aligned} &F_t(t,s) + rsF_s(t,s) \\ &\qquad + \frac{1}{2}s^2\sigma^2(t,s)F_{ss}(t,s) = 0, \\ &F(T,s) = \Phi(s). \end{aligned}$$

Björk points out that the above assumption of the derivative actually being traded on the market is a weak point. He therefore proceeds to show that the conclusions are independent of this fact by constructing a replicating portfolio that is based on S and B only.

Björk's book extends the theory in several directions. By using similar arguments as those for the Black–Scholes formula one can treat a whole zoo of different underlying stocks and derivatives, all widely used in the financial world.

There are forward contracts, for which the initial price is zero by definition (but which fetch a nonzero price until the exercise date), and conditions for the future commitment on both sides have to be specified. One may look at derivatives of underlying assets that are paying dividends, either at discrete times or continuously over time, or barrier options for which the payment of the claim is connected to the underlying stock reaching certain extremal values until the exercise date. A special case is currency derivatives and considerations of domestic and foreign equity markets. They are related to dividend paying derivatives as well as models with several underlying assets, which brings us to the next important section of the book.

It is shown that the Black–Scholes results may be generalized to claims that depend on several underlying assets, now modeling their prices by a system of coupled stochastic differential equations. However, we can only employ the above theory if the market is complete, which means that every (simple) claim can be replicated with a portfolio as described above. If the market model is further arbitrage free, we arrive again at a unique specification of the correct price for the derivative under question.

The study of completeness and the lack of arbitrage is summarized in the "metatheorem" that the chosen model will generally allow arbitrage possibilities if the number of assets exceeds the number of random sources in the model. Vice versa, if the number of assets, which we may use for hedging, is smaller than the number of random sources, then it can occur that not every claim can be replicated.

While a model with arbitrage possibilities is not very realistic, the assumption of a complete market is an idealized one itself. In an incomplete market a unique price cannot be specified by a replicating portfolio as before. Björk makes the point that in this situation there are several arbitrage-free pricing systems. Hence, we will have to turn back to the market for more information. Consistency conditions between the prices of various assets and their derivatives lead to a quantity, the "market price of risk," which is universal for all asset/derivative pairs. Observing the market price of risk for any such pair therefore specifies uniquely the price of the derivative under question.

Another section of the book is devoted to interest rate theory. Here, we are backing away from the assumption of a constant interest rate and are concerned with modeling the bond market. A zero coupon bond has a deterministic payoff at its maturity T; its price at time t is denoted by $p(t,T)$. As T may vary continuously, there are infinitely many such bonds. It is shown that the various kinds of coupon paying bonds, for which there are predetermined payoffs at certain times until maturity, can be expressed in terms of these.

Obvious questions to investigate are the relationships that must hold between the bonds of different maturities in order to assure an arbitrage-free bond market and, subsequently, the pricing of the bonds. It is clear from the above exposition that the second question cannot be answered without some observations from the market.

Spot rate models describe the dynamic of the spot rate $r(t) = p(t,t)$ by

$$dr = \mu(t,r)dt + \sigma(t,r)dW$$

and assume that $p(t,T) = F(t,r,T)$. But even then the market is not complete, and as for incomplete markets it is shown that knowledge of the evolution of one bond, specifically its market price of risk, dictates prices for all other bonds in an arbitrage-free market.

An alternative approach is forward rate models which use more than just one explanatory state variable. The famous Heath–Jarrow–Merton model focuses on the entire forward rate curve $f(t,T)$ given by

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW_t,$$

where W is a multidimensional Wiener process and

$$p(t,T) = \exp\left(-\int_t^T f(t,s)ds\right).$$

Here, conditions have to be found on the coefficients α and σ to ensure that the bond market is arbitrage free.

The final chapter of the book explains in detail how the correct price can be expressed in terms of the (discounted) expected value of a "risk neutral" or martingale measure. Under the martingale measure, which is equivalent to the objective measure (in a mathematical sense), calculations are generally easier and so the "change of numeraire" is an important tool.

Arbitrage Theory in Continuous Time is an excellent introductory book to some of the most important areas of the fast growing field of financial mathematics. Rather than plunging into the more technical depths of stochastic calculus the book gives a concise and mathematically precise outline and good intuition of the basic ideas of stochastic integration and differential equations. It also includes a self-contained introduction to stochastic optimal control.

The book clearly focuses on the economic motivation behind the models, thoroughly assessing the assumptions made as well as their implications, which are neither straightforward nor intuitive.

Due to this approach Björk's book makes a good read for mathematicians and economists alike. Mathematicians without any economics knowledge will appreciate that the crucial financial structures and terminologies are defined in a mathematical setting, while there is an abundance of explanations and examples highlighting their applications and importance in the financial world.

The brevity and simplifications of the mathematical arguments doesn't come at a great loss to the mathematician, as the stochastic calculus used is standard. The interested reader unfamiliar with the subject may fill in the details by consulting any of the numerous good texts on the topic. On the other hand, the economist with only a moderate mathematical background gets the chance to obtain a basic understanding of the mathematical theory, which will allow him or her to follow most arguments and derivations behind the famous formulae on an intuitive level.

Last but not least, the introductory level as well as the many examples and exercises make Björk's book a good textbook for mathematically oriented economists as well as probabilists interested in finance.

Anja Sturm
University of Oxford

Mechanics: From Newton's Laws to Deterministic Chaos. Third Edition. *By Florian Scheck.* Springer-Verlag, New York, 1999. $59.95. xiv+528 pp., softcover. ISBN 3-540-65558-1.

Reviewing a book with the title *Mechanics* one should first make precise what is understood by mechanics. Mechanics, certainly, is the oldest branch of physics dealing with the motion of bodies under the action of forces. However, there is a difference between physicists' and engineers' understandings of mechanics. Nowadays, for physicists, mechanics is a quite restricted field that only serves as a basis for more advanced topics in theoretical physics. The name *classical mechanics* would be more appropriate to this understanding (see also the titles of [1] and [2]). In books on classical mechanics the objects treated are particles and rigid bodies, with a short appendix occasionally added in which the transition of a multiparticle system to a continuum is performed. On the other hand, for engineers, mechanics means a much wider field of objects and concepts, where, in addition to the above-mentioned objects the focus is directed to the motion of deformable bodies in continuum mechanics. In continuum mechanics, for example, a great variety of diverse fields is treated, such as fluid dynamics, smart materials and structures, fracture and crack mechanics, viscoelasticity and creep, computational solids and fluid mechanics, to name a few.

Scheck's book belongs to the class of books on classical mechanics and presents the material taught to undergraduate students in physics at a major university. In an advanced formulation, as is given in this book, the main purpose is to present concepts and techniques, like action angle variables, the principle of least action, Hamilton–Jacobi theory, Poisson brackets, and canonical transformations, which are necessary for the treatment of quantum mechanics. However, the student learns to master these techniques while working in terms of the familiar concepts of classical mechanics. Moreover, the student learns to derive equations of motions from simple general principles, that is, to transform a physical problem into a mathematical equation.

The book under review is now appearing in its fifth German edition and its third English edition. Hence, there is no doubt that it has been well accepted by both students and teachers. This success is not so difficult to understand, because the author very skillfully manages to combine the content of two excellent classic books in this field, namely, [1] and [2]. Contrary to the widespread impression that classical mechanics is a complete, closed subject, there have been exciting new developments recently, best characterized by the terms *strange attractor* and *soliton*. Scheck also pays attention to these developments.

Whereas the content of [1] barely includes any treatment of the geometrical structure of mechanics and recent developments are only indicated verbally, the style and content of [2] in some of its chapters are too advanced for the undergraduate student. Scheck's intention is to fill this gap. In an extensive Chapter 5, geometric aspects of mechanics are treated, based on manifolds and differential forms. As an application, it is shown that Euler's equations of the rigid body are the geodesic equations of the Riemannian manifold of the three-dimensional rotation group. An extensive Chapter 6, titled "Stability and Chaos," is added, where an introduction to some concepts of nonlinear dynamics is given. In the short Chapter 7 the propagating soliton solution is calculated for the sine-Gordon equation, which is derived from a pendulum chain.

All these concepts are well explained and, hence, this is a very attractive textbook which can be handed over to any second or third year undergraduate in a physics department. Also very attractive is the large number of excercise problems (with extensive solutions), which will enable the student to check his or her level of understanding.

However, there are also some slight points of criticism. After the elegant treatment of the motion of a mass point in a moving frame in [2], one wonders why this has not yet become standard in recent textbooks. Moreover, the notation used in [2] in the derivation of Euler's equations of the rigid body is much more elegant than in the present text. In the presentation of the bifurcations of flows some improvement will be necessary in future editions of the book. For example, the essential reduction to the low-dimensional bifurcation system is indicated by the matrices **A** and **B** in (6.52) and (6.53) but should be better explained. One also wonders whether the definitions of chaos given are the best available for an introductory textbook.

If a student in physics asked me for advice on which book she or he should study to master a course in classical mechanics, I would not recommend reading only a single one. I would suggest starting with [1], then for the more advanced geometric theory, to try [2]. If there are problems understanding [2], the present book would be of great help.

REFERENCES

[1] H. Goldstein, *Classical Mechanics*, 2nd ed., Addison-Wesley, Reading, MA, 1980.

[2] V. I. Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, 1978.

Hans Troger
Vienna University of Technology

Matrix Analysis and Applied Linear Algebra. *By Carl D. Meyer.* SIAM, Philadelphia, 2000. $75.00. xii+718 pp., hardcover. ISBN 0-89871-454-0.

Carl Meyer's *Matrix Analysis and Applied Linear Algebra* is an introduction to the theory and practice of linear algebra for university students of mathematics, engineering, and science. Both theoretical and computational issues are addressed. Wherever possible, the concepts are illustrated by applications. The author takes a narrative, motivational approach, introducing most new concepts with simple examples or special cases. This does not mean that the theory gets shortchanged. Definitions and theorems are stated, but not without motivation, and the theorems are proved.

On the computational side, floating point arithmetic and roundoff errors are discussed, as are the need for pivoting in Gaussian elimination and the existence of ill-conditioned systems. The student is advised that the normal equations are not the best way to solve a least-squares problem and that one does not usually compute eigenvalues by solving the characteristic equation.

At the end of each section lies a good set of exercises ranging from routine to difficult. Many are straightforward computations. Others foreshadow coming developments, explore new applications, or fill in theoretical details.

Other noteworthy features: Numerous historical notes enrich the presentation. A solutions manual is available. Indeed, the book comes with a CD-ROM that contains not only the complete text, but also the solutions manual, biographical sketches of mathematicians, and a few other goodies, all in PDF format. There is a web site, www.matrixanalysis.com, at which the reader can access an errata list, replacement pages, and new material that will (presumably) appear in a later edition of the book.

The book contains much more material than can be covered in the standard one-semester introductory course. It is a delight to have such a large number of optional topics from which to pick and choose, but it is also a burden. I believe it will prove difficult to decide which important topics to include and which to omit, while still leaving enough time to get through all of the basic material. However, for those who have the luxury of teaching a year-long course, this will not be a problem.

Because of the large amount of material, the book can also be used for a more advanced course for students who have already had some exposure to linear algebra. The instructor can touch lightly on the basics and then plunge into the more advanced stuff, which includes, for example, the fast Fourier transform, the singular value decomposition, the Jordan canonical form, and Perron–Frobenius theory.

There were a few things I didn't like about the book. Here and there the motivation was left out. For example, floating point numbers are defined suddenly on page 21 with no preparatory examples or explanation whatsoever. Matters are made worse by an error in the definition: the exponent is confused with the number of digits in the exponent. (However, to be fair I must say that I found very few errors in the book.)

Section 3.4 is titled "Why Do It This Way," even though the *It* (matrix multiplication) and the *This Way* to which it refers have not yet been mentioned. The section is only one page long; it should have been the first page of the next section, which has the more reasonable title, "Matrix Multiplication."

While some sections are too short, quite a few others are too long, wandering from one topic to the next. If you want to look up practical methods for computing eigenvalues, you will not be able to find them by looking in the table of contents. You would never guess that the QR algorithm is discussed in section 7.3, which is titled "Functions of Diagonalizable Matrices." The pathway through section 7.3 is roughly this: diagonalization, spectral decomposition, analysis of Markov chains, power method, QR algorithm. There is a certain logic to this progression, but I think it would have made more sense to separate this material into two sections, especially since the connection between the power method and the QR algorithm is not explained nor even mentioned. (Instead the word "magic" is used.)

The book has no bibliography.

These shortcomings are not fatal. All in all, *Matrix Analysis and Applied Linear Algebra* is an excellent resource. In fact, I was surprised at how much I learned from reading it. If you are looking for a new linear algebra text, one that balances theory, computation, and applications, you should definitely take a look at this one.

DAVID S. WATKINS
Washington State University